HEAT TRANSFER

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1. Introduction

Energy is defined as the capacity of a substance to do work. It is a property of the substance and it can be transferred by interaction of a system and its surroundings. The student would have encountered these interactions during the study of Thermodynamics. However, Thermodynamics deals with the end states of the processes and provides no information on the physical mechanisms that caused the process to take place. Heat Transfer is an example of such a process. A convenient definition of heat transfer is energy in transition due to temperature differences. Heat transfer extends the Thermodynamic analysis by studying the fundamental processes and modes of heat transfer through the development of relations used to calculate its rate.

The aim of this chapter is to console existing understanding and to familiarise the student with the standard of notation and terminology used in this book. It will also introduce the necessary units.

1.1 Heat Transfer Modes

The different types of heat transfer are usually referred to as 'modes of heat transfer'. There are three of these: conduction, convection and radiation.

- Conduction: This occurs at molecular level when a temperature gradient exists in a medium, which can be solid or fluid. Heat is transferred along that temperature gradient by conduction.

- Convection: Happens in fluids in one of two mechanisms: random molecular motion which is termed diffusion or the bulk motion of a fluid carries energy from place to place. Convection can be either forced through for example pushing the flow along the surface or natural as that which happens due to buoyancy forces.

- Radiation: Occurs where heat energy is transferred by electromagnetic phenomenon, of which the sun is a particularly important source. It happens between surfaces at different temperatures even if there is no medium between them as long as they face each other.

In many practical problems, these three mechanisms combine to generate the total energy flow, but it is convenient to consider them separately at this introductory stage. We need to describe each process symbolically in an equation of reasonably simple form, which will provide the basis for subsequent calculations. We must also identify the properties of materials, and other system characteristics, that influence the transfer of heat.

1.2 System of Units

Before looking at the three distinct modes of transfer, it is appropriate to introduce some terms and units that apply to all of them. It is worth mentioning that we will be using the SI units throughout this book:

- The rate of heat flow will be denoted by the symbol Q. It is measured in Watts (W) and multiples such as (kW) and (MW).

- It is often convenient to specify the flow of energy as the heat flow per unit area which is also known as heat flux. This is denoted by q. Note that, $q = Q / A$ where A is the area through which the heat flows, and that the units of heat flux are (W/m2).

- Naturally, temperatures play a major part in the study of heat transfer. The symbol T will be used for temperature. In SI units, temperature is measured in Kelvin or Celsius: (K) and (°C). Sometimes the symbol t is used for temperature, but this is not appropriate in the context of transient heat transfer, where it is convenient to use that symbol for time. Temperature difference is denoted in Kelvin (K).

The following three subsections describe the above mentioned three modes of heat flow in more detail. Further details of conduction, convection and radiation will be presented in Chapters 2, 3 and 4 respectively. Chapter 5 gives a brief overview of Heat Exchangers theory and application which draws on the work from the previous Chapters.

1.3 Conduction

The conductive transfer is of immediate interest through solid materials. However, conduction within fluids is also important as it is one of the mechanisms by which heat reaches and leaves the surface of a solid. Moreover, the tiny voids within some solid materials contain gases that conduct heat, albeit not very effectively unless they are replaced by liquids, an event which is not uncommon. Provided that a fluid is still or very slowly moving, the following analysis for solids is also applicable to conductive heat flow through a fluid.

Figure 1.1 shows, in schematic form, a process of conductive heat transfer and identifies the key quantities to be considered:

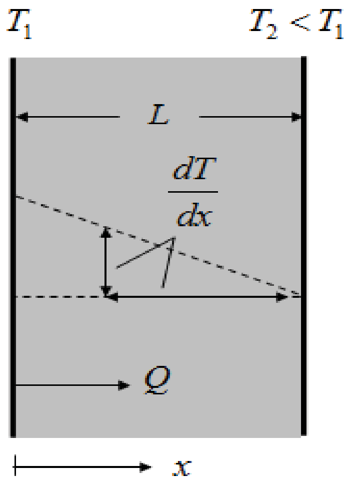

Figure 1-1: One dimensional conduction

Q : the heat flow by conduction in the x-direction (W)

A : the area through which the heat flows, normal to the x-direction (m2)

$\frac{dT}{dx}$: the temperature gradient in the x-direction (K/m)

These quantities are related by Fourier's Law, a model proposed as early as 1822:

$$Q = -k A \frac{dT}{dx} \quad \text{or} \quad q = -k \frac{dT}{dx} \quad 1 \qquad (1.1)$$

A significant feature of this equation is the negative sign. This recognises that the natural direction for the flow of heat is from high temperature to low temperature, and hence down the temperature gradient.

The additional quantity that appears in this relationship is k , the thermal conductivity (W/m K) of the material through which the heat flows. This is a property of the particular heat-conducting substance and, like other properties, depends on the state of the material, which is usually specified by its temperature and pressure.

The dependence on temperature is of particular importance. Moreover, some materials such as those used in building construction are capable of absorbing water, either in finite pores or at the molecular level, and the moisture content also influences the thermal conductivity. The units of thermal conductivity have been determined from the requirement that Fourier's law must be dimensionally consistent.

Considering the finite slab of material shown in Figure 1.1, we see that for one-dimensional conduction the temperature gradient is:

$$\frac{dT}{dx} = \frac{T_2 - T_1}{L}$$

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Hence for this situation the transfer law can also be written

$$Q = k A \frac{T_1 - T_2}{L} \quad \text{or} \quad q = k \frac{T_1 - T_2}{L} \quad 3 \qquad (1.2)$$

$$\alpha = \frac{k}{\rho \; C} \qquad (1.3)4$$

Table 1.1 gives the values of thermal conductivity of some representative solid materials, for conditions of normal temperature and pressure. Also shown are values of another property characterising the flow of heat through materials, thermal diffusivity, which is related to the conductivity by:

Where ρ is the density in kg/m^3 of the material and C its specific heat capacity in $J/kg\,K$.

The thermal diffusivity indicates the ability of a material to transfer thermal energy relative to its ability to store it. The diffusivity plays an important role in unsteady conduction, which will be considered in Chapter 2.

As was noted above, the value of thermal conductivity varies significantly with temperature, even over the range of climatic conditions found around the world, let alone in the more extreme conditions of cold-storage plants, space flight and combustion. For solids, this is illustrated by the case of mineral wool, for which the thermal conductivity might change from 0.04 to 0.28 W/m K across the range 35 to - 35 °C.

Table 1-1 Thermal conductivity and diffusivity for typical solid materials at room temperature

Material	k W/m K	α mm2/s	Material	k W/m K	α mm2/s
Copper	350	115	Medium concrete block	0.5	0.35
Aluminium	236	85	Dense plaster	0.5	0.40
Mild steel	50	13	Stainless steel	14	4
Polyethylene	0.5	0.15	Nylon, Rubber	0.25	0.10
Face Brick	1.0	0.75	Aerated concrete	0.15	0.40
Glass	0.9	0.60	Wood, Plywood	0.15	0.2
Fireclay brick	1.7	0.7	Wood-wool slab	0.10	0.2
Dense concrete	1.4	0.8	Mineral wool expanded	0.04	1.2
Common brick	0.6	0.45	Expanded polystyrene	0.035	1.0

For gases the thermal conductivities can vary significantly with both pressure and temperature. For liquids, the conductivity is more or less insensitive to pressure. Table 1.2 shows the thermal conductivities for typical gases and liquids at some given conditions.

Table 1-2 Thermal conductivity for typical gases and liquids

Material	k [W/m K]
Gases	
Argon (at 300 K and 1 bar)	0.018
Air (at 300 K and 1 bar)	0.026
Air (at 400 K and 1 bar)	0.034
Hydrogen (at 300 K and 1 bar)	0.180
Freon 12 (at 300 K 1 bar)	0.070
Liquids	
Engine oil (at 20oC)	0.145
Engine oil (at 80oC)	0.138
Water (at 20oC)	0.603
Water (at 80oC)	0.670

Mercury(at 27oC)	8.540

Note the very wide range of conductivities encountered in the materials listed in Tables 1.1 and 1.2. Some part of the variability can be ascribed to the density of the materials, but this is not the whole story (Steel is more dense than aluminium, brick is more dense than water). Metals are excellent conductors of heat as well as electricity, as a consequence of the free electrons within their atomic lattices. Gases are poor conductors, although their conductivity rises with temperature (the molecules then move about more vigorously) and with pressure (there is then a higher density of energy-carrying molecules). Liquids, and notably water, have conductivities of intermediate magnitude, not very different from those for plastics. The low conductivity of many insulating materials can be attributed to the trapping of small pockets of a gas, often air, within a solid material which is itself a rather poor conductor.

Example 1.1

Calculate the heat conducted through a 0.2 m thick industrial furnace wall made of fireclay brick. Measurements made during steady-state operation showed that the wall temperatures inside and outside the furnace are 1500 and 1100 K respectively. The length of the wall is 1.2m and the height is 1m.

Solution

We first need to make an assumption that the heat conduction through the wall is one dimensional. Then we can use Equation 1.2:

$$Q = k\,A\,\frac{T_2 - T_1}{L}$$

The thermal conductivity for fireclay brick obtained from Table 1.1 is 1.7 W/m K

The area of the wall $A = 1.2 \times 1.0 = 1.2 \quad \mathrm{m}^2$

Thus:

$$Q = 1.7\ \mathrm{W/m\,K} \times 1.2\ \mathrm{m}^2 \times \frac{1500\ \mathrm{K} - 1100\ \mathrm{K}}{0.2\ \mathrm{m}} = 4080\ \mathrm{W}$$

Comment: Note that the direction of heat flow is from the higher temperature inside to the lower temperature outside.

1.4 Convection

Convection heat transfer occurs both due to molecular motion and bulk fluid motion. Convective heat transfer may be categorised into two forms according to the nature of the flow: natural Convection and forced convection.

In natural of ‘free’ convection, the fluid motion is driven by density differences associated with temperature changes generated by heating or cooling. In other words, fluid flow is induced by buoyancy forces. Thus the heat transfer itself generates the flow which conveys energy away from the point at which the transfer occurs.

In forced convection, the fluid motion is driven by some external influence. Examples are the flows of air induced by a fan, by the wind, or by the motion of a vehicle, and the flows of water within heating, cooling, supply and drainage systems. In all of these processes the moving fluid conveys energy, whether by design or inadvertently.

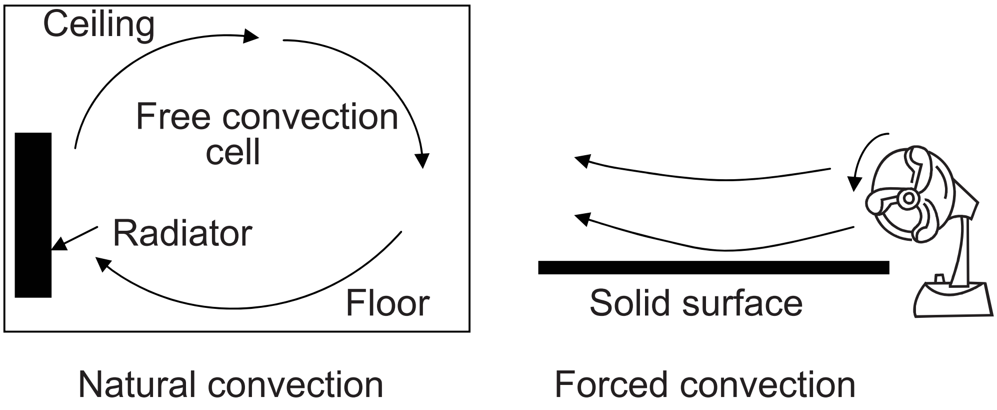

Figure 1-2: Illustration of the process of convective heat transfer

The left of Figure 1.2 illustrates the process of natural convective heat transfer. Heat flows from the 'radiator' to the adjacent air, which then rises, being lighter than the general body of air in the room. This air is replaced by cooler, somewhat denser air drawn along the floor towards the radiator. The rising air flows along the ceiling, to which it can transfer heat, and then back to the lower part of the room to be recirculated through the buoyancy-driven 'cell' of natural convection.

The word 'radiator' has been written above in that way because the heat transfer from such devices is not predominantly through radiation; convection is important as well. In fact, in a typical central heating radiator approximately half the heat transfer is by (free) convection.

The right part of Figure 1.2 illustrates a process of forced convection. Air is forced by a fan carrying with it heat from the wall if the wall temperature is lower or giving heat to the wall if the wall temperature is lower than the air temperature.

If T_1 is the temperature of the surface receiving or giving heat, and T_∞ is the average temperature of the stream of fluid adjacent to the surface, then the convective heat transfer Q is governed by Newton's law:

$$Q = h_c A (T_1 - T_2) \quad \text{or} \quad q = h_c \, (T_1 - T_2) \; 5 \quad (1.3)$$

Another empirical quantity has been introduced to characterise the convective transfer mechanism. This is hc, the convective heat transfer coefficient, which has units [W/m2 K].

This quantity is also known as the convective conductance and as the film coefficient. The term film coefficient arises from a simple, but not entirely unrealistic, picture of the process of convective heat transfer at a surface. Heat is imagined to be conducted through a thin stagnant film of fluid at the surface, and then to be convected away by the moving fluid beyond. Since the fluid right against the wall must actually be at rest, this is a fairly reasonable model, and it explains why convective coefficients often depend quite strongly on the conductivity of the fluid.

Table 1-3 Representative range of convective heat transfer coefficient

Nature of Flow	Fluid	hc [W/m2 K]
Surfaces in buildings	Air	1 - 5
Surfaces outside buildings	Air	5-150
Across tubes	Gas	10 - 60
	Liquid	60 - 600
In tubes	Gas	60 - 600
	Organic liquid	300 - 3000
	Water	600 - 6000
	Liquid metal	6000 - 30000
Natural convection	Gas	0.6 - 600
	Liquid	60 - 3000
Condensing	Liquid film	1000 - 30000
	Liquid drops	30000 - 300000
Boiling	Liquid/vapour	1000 - 10000

The film coefficient is not a property of the fluid, although it does depend on a number of fluid properties: thermal conductivity, density, specific heat and viscosity. This single quantity subsumes a variety of features of the flow, as well as characteristics of the convecting fluid. Obviously, the velocity of the flow past the wall is significant, as is the fundamental nature of the motion, that is to say, whether it is turbulent or laminar. Generally speaking, the convective coefficient increases as the velocity increases.

A great deal of work has been done in measuring and predicting convective heat transfer coefficients. Nevertheless, for all but the simplest situations we must rely upon empirical data, although numerical methods based on computational fluid dynamics (CFD) are becoming increasingly used to compute the heat transfer coefficient for complex situations.

Table 1.3 gives some typical values; the cases considered include many of the situations that arise within buildings and in equipment installed in buildings.

Example 1.2

A refrigerator stands in a room where the air temperature is 20oC. The surface temperature on the outside of the refrigerator is 16oC. The sides are 30 mm thick and have an equivalent thermal conductivity of 0.1 W/m K. The heat transfer coefficient on the outside 9 is 10 W/m2K. Assuming one dimensional conduction through the sides, calculate the net heat flow and the surface temperature on the inside.

Solution

Let $T_{s,i}$ and $T_{s,o}$ be the inside surface and outside surface temperatures, respectively and T_f the fluid temperature outside.

The rate of heat convection per unit area can be calculated from Equation 1.3:

$$q = h(T_{s,o} - T_f)$$

$$q = 10 \times (16 - 20) = -40 \text{ W/m}^2$$

This must equal the heat conducted through the sides. Thus we can use Equation 1.2 to calculate the surface temperature:

$$q = -k \frac{T_{s,o} - T_{s,i}}{L}$$

$$-40 = -0.1 \times \frac{16 - T_{s,i}}{0.03}$$

$$T_{s,i} = 4°C$$

Comment: This example demonstrates the combination of conduction and convection heat transfer relations to establish the desired quantities.

1.5 Radiation

While both conductive and convective transfers involve the flow of energy through a solid or fluid substance, no medium is required to achieve radiative heat transfer. Indeed, electromagnetic radiation travels most efficiently through a vacuum, though it is able to pass quite effectively through many gases, liquids and through some solids, in particular, relatively thin layers of glass and transparent plastics.

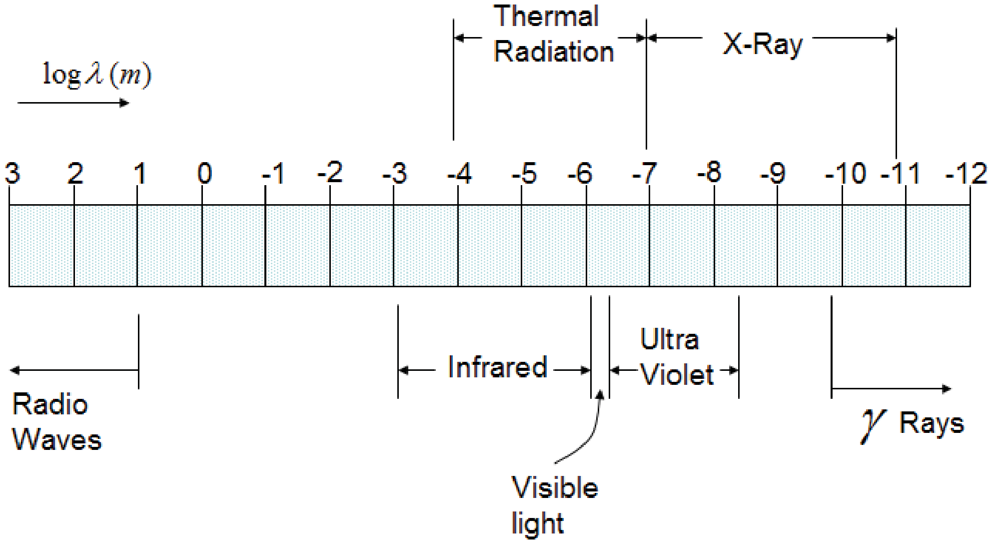

Figure 1-3: Illustration of electromagnetic spectrum

Figure 1.3 indicates the names applied to particular sections of the electromagnetic spectrum where the band of thermal radiation is also shown. This includes:

- the rather narrow band of visible light;
- the wider span of thermal radiation, extending well beyond the visible spectrum.

Our immediate interest is thermal radiation. It is of the same family as visible light and behaves in the same general fashion, being reflected, refracted and absorbed. These phenomena are of particular importance in the calculation of solar gains, the heat inputs to buildings from the sun and radiative heat transfer within combustion chambers.

It is vital to realise that every body, unless at the absolute zero of temperature, both emits and absorbs energy by radiation. In many circumstances the inwards and outwards transfers nearly cancel out, because the body is at about the same temperature as its surroundings. This is your situation as you sit reading these words, continually exchanging energy with the surfaces surrounding you.

In l884 Boltzmann put forward an expression for the net transfer from an idealised body (Black body) with surface area A1 at absolute temperature T1 to surroundings at uniform absolute temperature T2:

$$Q = \sigma A_1 (T_1^4 - T_2^4) \quad \text{or} \quad q = \sigma (T_1^4 - T_2^4) \qquad (1.4)$$

with σ the Stefan-Boltzmann constant, which has the value 5.67 x 10-8 W/m2 K4 and T [K] = T [°C] + 273 is the absolute temperature.

The bodies considered above are idealised, in that they perfectly absorb and emit radiation of all wave-lengths. The situation is also idealised in that each of the bodies that exchange radiation has a uniform surface temperature. A development of Boltzmann's law which allows for deviations from this pattern is

$$Q = \varepsilon \sigma F_{12} A_1 (T_1^4 - T_2^4) \qquad 7 \; (1.5)$$

With ε the emissivity, or emittance, of the surface A1, a dimensionless factor in the range 0 to 1,

F12 is the view factor, or angle factor, giving the fraction of the radiation from A1 that falls on the area A2 at temperature T2, and therefore also in the range 0 to 1.

Another property of the surface is implicit in this relationship: its absorbtivity. This has been taken to be equal to the emissivity. This is not always realistic. For example, a surface receiving short-wave-length radiation from the sun may reject some of that energy by re-radiation in a lower band of wave-lengths, for which the emissivity is different from the absorbtivity for the wave-lengths received.

The case of solar radiation provides an interesting application of this equation. The view factor for the Sun, as seen from the Earth, is very small; despite this, the very high solar temperature (raised to the power 4) ensures that the radiative transfer is substantial. Of course, if two surfaces do not 'see' one another (as, for instance, when the Sun is on the other side of the Earth), the view factor is zero. Table 1.4 shows values of the emissivity of a variety of materials. Once again we find that a wide range of characteristics are available to the designer who seeks to control heat transfers.

The values quoted in the table are averages over a range of radiation wave-lengths. For most materials, considerable variations occur across the spectrum. Indeed, the surfaces used in solar collectors are chosen because they possess this characteristic to a marked degree. The emissivity depends also on temperature, with the consequence that the radiative heat transfer is not exactly proportional to T3.

An ideal emitter and absorber is referred to as a 'black body', while a surface with an emissivity less than unity is referred to as 'grey'. These are somewhat misleading terms, for our interest here is in the infra-red spectrum rather than the visible part. The appearance of a surface to the eye may not tell us much about its heat-absorbing characteristics.

Table 1-4 Representative values of emissivity

Ideal 'black' body	1.00	Aluminium paint	0.5
White paint	0.97	Galvanised steel	0.3
Gloss paint	0.9	Stainless steel	0.15
Brick	0.9	Aluminium foil	0.12
Rusted steel	0.8	Polished copper	0.03
		Perfect mirror	0

Although it depends upon a difference in temperature, Boltzmann's Law (Equations 1.4, 1.5) does not have the precise form of the laws for conductive and convective transfers. Nevertheless, we can make the radiation law look like the others. We introduce a radiative heat transfer coefficient or radiative conductance through

$$Q = h_r A_1 (T_1 - T_2) \quad 8 \quad (1.6)$$

Comparison with the developed form of the Boltzmann Equation (1.5), plus a little algebra, gives

$$h_r = \frac{Q}{A_1 (T_1 - T_2)} = \varepsilon \sigma F_{12} (T_1 + T_2)(T_1^2 + T_2^2)$$

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If the temperatures of the energy-exchanging bodies are not too different, this can be approximated by

$$h_r = 4 \varepsilon \sigma F_{12} T_{av}^3 \quad 10 \quad (1.7)$$

where Tav is the average of the two temperatures.

Obviously, this simplification is not applicable to the case of solar radiation. However, the temperatures of the walls, floor and ceiling of a room generally differ by only a few degrees. Hence the approximation given by Equation (1.7) is adequate when transfers between them are to be calculated.

Example 1.3

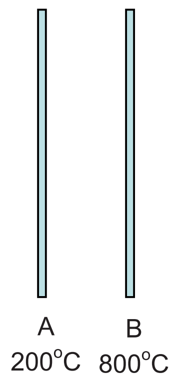
Surface A in the Figure is coated with white paint and is maintained at temperature of 200oC. It is located directly opposite to surface B which can be considered a black body and is maintained ate temperature of 800oC. Calculate the amount of heat that needs to be removed from surface A per unit area to maintain its constant temperature.

Solution

The two surfaces are assumed to be infinite and close to each other that they are only exchanging heat with each other. The view factor can then assumed to be 1.

The heat gained by surface A by radiation from surface B can be computed from Equation 1.5:

$$q = \varepsilon \sigma F_{AB} (T_B^4 - T_A^4)$$



The emissivity of white coated paint is 0.97 from Table 1.4

Thus

$$q = 0.97 \times 5.67 \times 10^{-8} \times 1 (1073^4 - 400^4) = 71469 \text{ W/m}^2$$

This amount of heat needs to be removed from surface A by other means such as conduction, convection or radiation to other surfaces to maintain its constant temperature.

1.6 Summary

This chapter introduced some of the basic concepts of heat transfer and indicates their significance in the context of engineering applications.

We have seen that heat transfer can occur by one of three modes, conduction, convection and radiation. These often act together. We have also described the heat transfer in the three forms using basic laws as follows:

Conduction: $Q = -kA\frac{dT}{dx}$ [W]

Where thermal conductivity k [W/m K] is a property of the material

Convection from a surface: $Q = hA(T_s - T_\infty)$ [W]

Where the convective coefficient h [W/m2 K] depends on the fluid properties and motion.

Radiation heat exchange between two surfaces of temperatures T_1 and T_2:

$$Q = \varepsilon \sigma F_{12} A_1 (T_1^4 - T_2^4)$$

Where ε is the Emissivity of surface 1 and F12 is the view factor.

Typical values of the relevant material properties and heat transfer coefficients have been indicated for common materials used in engineering applications.

1.7 Multiple Choice assessment

1. The units of heat flux are:
   - Watts
   - Joules
   - Joules / meters2
   - Watts / meters2
   - Joules / Kg K

2. The units of thermal conductivity are:
   - Watts / meters2 K
   - Joules
   - Joules / meters2
   - Joules / second meter K
   - Joules / Kg K

3. The heat transfer coefficient is defined by the relationship
    - $h = m\ Cp\ \Delta T$
    - $h = k / L$
    - $h = q / \Delta T$
    - $h = Nu\ k / L$
    - $h = Q / \Delta T$

4. Which of these materials has the highest thermal conductivity ?
    - air
    - water
    - mild steel
    - titanium
    - aluminium

5. Which of these materials has the lowest thermal conductivity ?
    - air
    - water
    - mild steel
    - titanium
    - aluminium

6. In which of these is free convection the dominant mechanism of heat transfer ?
    - heat transfer to a piston head in a diesel engine combustion chamber
    - heat transfer from the inside of a fan-cooled p.c.
    - heat transfer to a solar heating panel
    - heat transfer on the inside of a central heating panel radiator
    - heat transfer on the outside of a central heating panel radiator

7. Which of these statements is not true ?
    - conduction can occur in liquids
    - conduction only occurs in solids
    - thermal radiation can travel through empty space
    - convection cannot occur in solids
    - gases do not absorb thermal radiation

8. What is the heat flow through a brick wall of area 10m2, thickness 0.2m, k = 0.1 W/m K with a surface temperature on one side of 20ºC and 10ºC on the other ?
    - 50 Watts
    - 50 Joules
    - 50 Watts / m2
    - 200 Watts
    - 200 Watts / m2

9. The governing equations of fluid motion are known as:
    - Maxwell's equations
    - C.F.D.
    - Reynolds – Stress equations
    - Lame's equations
    - Navier – Stokes equations

10. A pipe of surface area 2m2 has a surface temperature of 100ºC, the adjacent fluid is at 20ºC, the heat transfer coefficient acting between the two is 20 W/m2K. What is the heat flow by convection ?
    - 1600 W
    - 3200 W
    - 20 W
    - 40 W
    - zero

11. The value of the Stefan-Boltzmann constant is:
    - 56.7 x 10-6 W/m2K4
    - 56.7 x 10-9 W/m2K4
    - 56.7 x 10-6 W/m2K
    - 56.7 x 10-9 W/m2K
    - 56.7 x 10-6 W/m K

12. Which of the following statements is true: Heat transfer by radiation ….
    - only occurs in outer space
    - is negligible in free convection
    - is a fluid phenomenon and travels at the speed of the fluid
    - is an acoustic phenomenon and travels at the speed of sound
    - is an electromagnetic phenomenon and travels at the speed of light

13. Calculate the net thermal radiation heat transfer between two surfaces. Surface A, has a temperature of 100ºC and Surface B, 200ºC. Assume they are sufficiently close so that all the radiation leaving A is intercepted by B and vice-versa. Assume also black-body behaviour.
    - 85 W
    - 85 W / m2
    - 1740 W
    - 1740 W / m2
    - none of these

14. The different modes of heat transfer are:
    - forced convection, free convection and mixed convection
    - conduction, radiation and convection
    - laminar and turbulent
    - evaporation, condensation and boiling
    - cryogenic, ambient and high temperature

15. Mixed convection refers to:
    - combined convection and radiation
    - combined convection and conduction
    - combined laminar and turbulent flow
    - combined forced and free convection
    - combined forced convection and conduction

16. The thermal diffusivity, α, is defined as:
    - $= \mu\ Cp / k$
    - $= k\ Cp / \rho$
    - $= k / \rho\ Cp$
    - $= h\ L / k$
    - $= L / k$

2. Conduction

2.1 The General Conduction Equation

Conduction occurs in a stationary medium which is most likely to be a solid, but conduction can also occur in fluids. Heat is transferred by conduction due to motion of free electrons in metals or atoms in non-metals. Conduction is quantified by Fourier's law: the heat flux, q, is proportional to the temperature gradient in the direction of the outward normal. e.g. in the x-direction:

$$q_x \propto \frac{dT}{dx} \qquad (2.1)$$

$$q_x = -k\frac{dT}{dx} \qquad (W/m^2) \qquad (2.2)$$

The constant of proportionality, k is the thermal conductivity and over an area A, the rate of heat flow in the x-direction, Q_x is

$$Q_x = -k\ A\frac{dT}{dx} \qquad (W) \qquad (2.3)$$

Conduction may be treated as either steady state, where the temperature at a point is constant with time, or as time dependent (or transient) where temperature varies with time.

The general, time dependent and multi-dimensional, governing equation for conduction can be derived from an energy balance on an element of dimensions $\delta x, \delta y, \delta z$.

Consider the element shown in Figure 2.1. The statement of energy conservation applied to this element in a time period δt is that:

heat flow in + internal heat generation = heat flow out + rate of increase in internal energy

$$Q_x + Q_y + Q_z + Q_g = Q_{x+\delta x} + Q_{y+\delta y} + Q_{z+\delta z} + mC\frac{\partial T}{\partial t} \qquad (2.4)$$

or

$$Q_x - Q_{x+\delta x} + Q_y - Q_{y+\delta y} + Q_z - Q_{z+\delta z} + Q_g + mC\frac{\partial T}{\partial t} = 0 \qquad (2.5)$$

As noted above, the heat flow is related to temperature gradient through Fourier's Law, so:

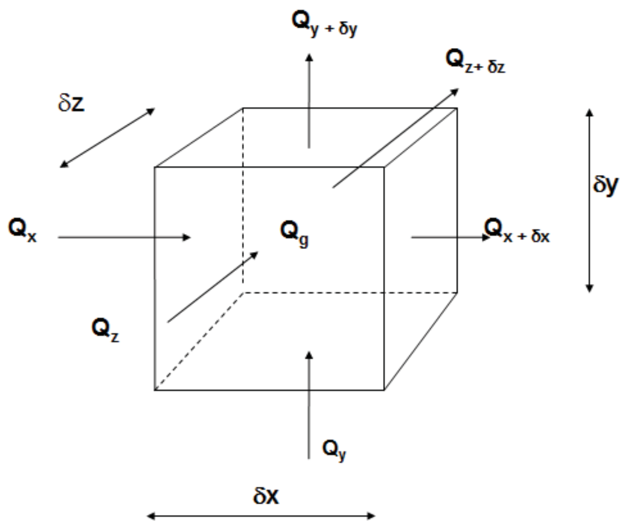


Figure 2-1 Heat Balance for conduction in an infinitismal element

$$Q_x = -k\,A\frac{dT}{dx} = -k\,\delta y\,\delta z\frac{dT}{dx}$$

$$Q_y = -k\,A\frac{dT}{dy} = -k\,\delta x\,\delta z\frac{dT}{dy} \qquad (2.6)$$

$$Q_z = -k\,A\frac{dT}{dz} = -k\,\delta x\,\delta y\frac{dT}{dx}$$

Using a Taylor series expansion:

$$Q_{x+\delta x} = Q_x + \frac{\partial Q_x}{\partial x}\delta x + \frac{1}{2!}\frac{\partial^2 Q_x}{\partial x^2}\delta x^2 + \frac{1}{3!}\frac{\partial^3 Q_x}{\partial x^3}\delta x^3 + \ldots \qquad (2.7)$$

For small values of δx it is a good approximation to ignore terms containing δx2 and higher order terms, So:

$$Q_x - Q_{x+\delta x} \cong \frac{\partial Q_x}{\partial x}\delta x = \delta x \delta y \delta z \frac{\partial}{\partial x}\left(-k\frac{\partial T}{\partial x}\right) \qquad (2.8)$$

A similar treatment can be applied to the other terms. For time dependent conduction in three dimensions (x,y,z), with internal heat generation $q_g(W/m^3) = Q_g/\delta x\,\delta y\,\delta z$:

$$\frac{\partial}{\partial x}\left(-k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(-k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(-k\frac{\partial T}{\partial z}\right) + q_g = \rho C\frac{\partial T}{\partial t} \qquad (2.9)$$

For constant thermal conductivity and no internal heat generation (Fourier's Equation):

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{\rho C}{k}\frac{\partial T}{\partial t} \qquad (2.10)$$

Where $(k/\rho C)$ is known as α , the thermal diffusivity (m2/s).

For steady state conduction with constant thermal conductivity and no internal heat generation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \qquad (2.11)$$

Similar governing equations exist for other co-ordinate systems. For example, for 2D cylindrical coordinate system (r, z,). In this system there is an extra term involving 1/r which accounts for the variation in area with r.

$$\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = 0 \qquad (2.12)$$

For one-dimensional steady state conduction (in say the x-direction)

$$\frac{d^2 T}{dx^2} = 0 \qquad (2.13)$$

It is possible to derive analytical solutions to the 2D (and in some cases 3D) conduction equations. However, since this is beyond the scope of this text the interested reader is referred to the classic text by Carlslaw and Jaeger (1980)

A meaningful solution to one of the above conduction equations is not possible without information about what happens at the boundaries (which usually coincide with a solid-fluid or solid-solid interface). This information is known as the boundary conditions and in conduction work there are three main types:

1. where temperature is specified, for example the temperature of the surface of a turbine disc, this is known as a boundary condition of the 1st kind;

2. where the heat flux is specified, for example the heat flux from a power transistor to its heat sink, this is known as a boundary condition of the 2nd kind;

3. where the heat transfer coefficient is specified, for example the heat transfer coefficient acting on a heat exchanger fin, this is known as a boundary condition of the 3rd kind.

2.1.1 Dimensionless Groups for Conduction

There are two principal dimensionless groups used in conduction. These are: The Biot number, $Bi = hL/k$ and The Fourier number, $Fo = \alpha t / L^2$.

It is customary to take the characteristic length scale L as the ratio of the volume to exposed surface area of the solid.

The Biot number can be thought of as the ratio of the thermal resistance due to conduction (L/k) to the thermal resistance due to convection 1/h. So for Bi << 1, temperature gradients within the solid are negligible and for Bi > 1 they are not. The Fourier number can be thought of as a time constant for conduction. For $Fo < 1$, time dependent effects are significant and for $Fo >> 1$ they are not.

2.1.2 One-Dimensional Steady State Conduction in Plane Walls

In general, conduction is multi-dimensional. However, we can usually simplify the problem to two or even one-dimensional conduction. For one-dimensional steady state conduction (in say the x-direction):

$$\frac{d^2T}{dx^2} = 0 \qquad (2.14)$$

From integrating twice:

$$T = C_1 x + C_2$$

where the constants C1 and C2 are determined from the boundary conditions. For example if the temperature is specified (1st Kind) on one boundary T = T1 at x = 0 and there is convection into a surrounding fluid (3rd Kind) at the other boundary $-k(dT/dx) = h(T_2 - T_{fluid})$ at x = L then:

$$T = T_1 - \left\{ \frac{hx}{k} \left(T_2 - T_{fluid} \right) \right\} \qquad (2.15)$$

which is an equation for a straight line.

To analyse 1-D conduction problems for a plane wall write down equations for the heat flux q. For example, the heat flows through a boiler wall with convection on the outside and convection on the inside:

$$q = h_{inside}(T_{inside} - T_1)$$

$$q = (k / L)(T_1 - T_2)$$

$$q = h_{outside}(T_2 - T_{outside})$$

Rearrange, and add to eliminate T1 and T2 (wall temperatures)

$$q = \frac{T_{inside} - T_{otside}}{\left(\frac{1}{h_{inside}}\right) + \left(\frac{1}{k}\right) + \left(\frac{1}{h_{outside}}\right)} \quad (2.16)$$

Note the similarity between the above equation with I = V / R (heat flux is the analogue of electrical current, temperature is of voltage and the denominator is the overall thermal resistance, comprising individual resistance terms from convection and conduction.

In building services it is common to quote a 'U' value for double glazing and building heat loss calculations. This is called the overall heat transfer coefficient and is the inverse of the overall thermal resistance.

$$U = \frac{1}{\left(\frac{1}{h_{inside}}\right) + \left(\frac{1}{k}\right) + \left(\frac{1}{h_{outside}}\right)} \quad (2.17)$$

2.1.3 The Composite Plane Wall

The extension of the above to a composite wall (Region 1 of width L1, thermal conductivity k1, Region 2 of width L2 and thermal conductivity k2 . . . . etc. is fairly straightforward.

$$q = \frac{T_{inside} - T_{otside}}{\left(\frac{1}{h_{inside}}\right) + \left(\frac{L_1}{k_1}\right) + \left(\frac{L_2}{k_2}\right) + \left(\frac{L_3}{k_3}\right) + \left(\frac{1}{h_{outside}}\right)} \quad (2.18)$$

Example 2.1

The walls of the houses in a new estate are to be constructed using a ‘cavity wall’ design. This comprises an inner layer of brick (k = 0.5 W/m K and 120 mm thick), an air gap and an outer layer of brick (k = 0.3 W/m K and 120 mm thick). At the design condition the inside room temperature is 20ºC, the outside air temperature is -10ºC; the heat transfer coefficient on the inside is 10 W/m2 K, that on the outside 40 W/m2 K, and that in the air gap 6 W/m2 K. What is the heat flux through the wall?

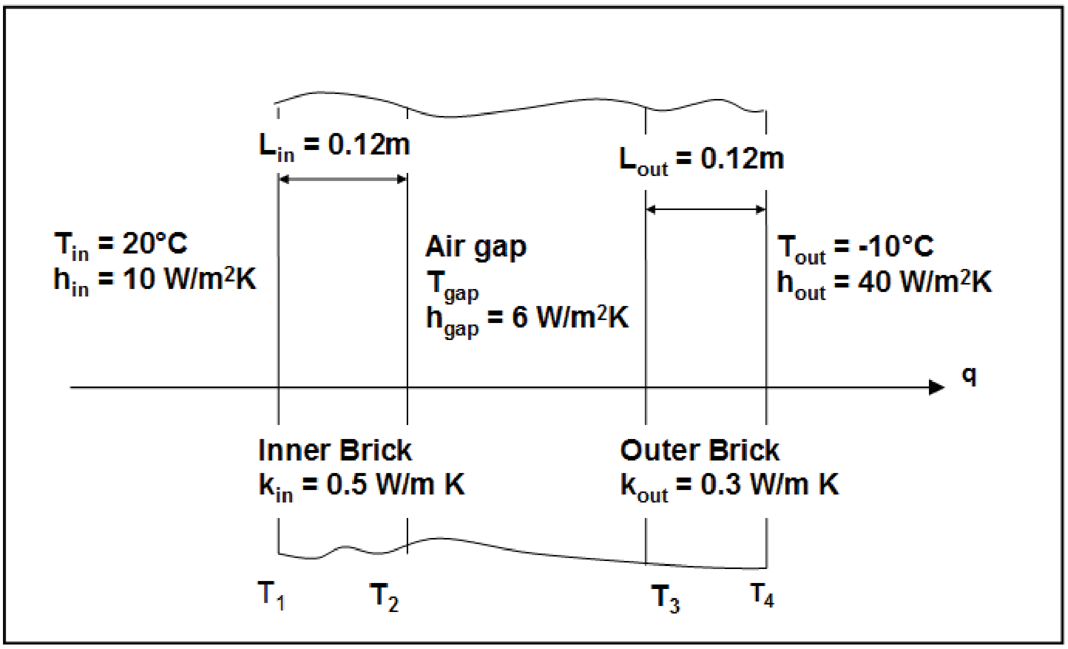


Figure 2-2: Conduction through a plane wall

Note the arrow showing the heat flux which is constant through the wall. This is a useful concept, because we can simply write down the equations for this heat flux.

Convection from inside air to the surface of the inner layer of brick

$$q = h_{in}(T_{in} - T_1)$$

Conduction through the inner layer of brick

$$q = k_{in} / L_{in}(T_1 - T_2)$$

Convection from the surface of the inner layer of brick to the air gap

$$q = h_{gap}(T_2 - T_{gap})$$

Convection from air gap to the surface of the outer layer of brick

$$q = h_{gap}(T_{gap} - T_3)$$

Conduction through the outer layer of brick

$$q = k_{out} / L_{out}(T_3 - T_4)$$

Convection from the surface of the outer layer of brick to the outside air

$$q = h_{out}(T_4 - T_{out})$$

The above provides six equations with six unknowns (the five temperatures T1, T2, T3, T4 and T_{gap} and the heat flux q). They can be solved simply by rearranging with the temperatures on the left hand side.

$$(T_{in} - T_1) = q / h_{in}$$

$$(T_2 - T_1) = q / (k_{in} / L_{in})$$

$$(T_2 - T_{gap}) = q / h_{gap}$$

$$(T_{gap} - T_3) = q / h_{gap}$$

$$(T_3 - T_4) = q / (k_{out} / L_{out})$$

$$(T_4 - T_{out}) = q / h_{out}$$

Then by adding, the unknown temperatures are eliminated and the heat flux can be found directly

$$q = \frac{T_{in} - T_{out}}{\left(\frac{1}{h_{in}}\right) + \left(\frac{L_{in}}{k_{in}}\right) + \left(\frac{1}{h_{gap}}\right) + \left(\frac{1}{h_{gap}}\right) + \left(\frac{L_{out}}{k_{out}}\right) + \left(\frac{1}{h_{out}}\right)}$$

$$q = \frac{20 - (-10)}{\left(\frac{1}{10}\right) + \left(\frac{0.12}{0.5}\right) + \left(\frac{1}{6}\right) + \left(\frac{1}{6}\right) + \left(\frac{0.12}{0.3}\right) + \left(\frac{1}{40}\right)} = 27.3 W / m^2$$

It is instructive to write out the separate terms in the denominator as it can be seen that the greatest thermal resistance is provided by the outer layer of brick and the least thermal resistance by convection on the outside surface of the wall. Once the heat flux is known it is a simple matter to use this to find each of the surface temperatures. For example,

$$T_4 = (q / h_{out}) + T_{out}$$

$$T_4 = (27.3 / 40) - 10$$

$$T_4 = -9.32°C$$

Thermal Contact Resistance

In practice when two solid surfaces meet then there is not perfect thermal contact between them. This can be accounted for using an appropriate value of thermal contact resistance – which can be obtained either from experimental results or published, tabulated data.

2.2 One-Dimensional Steady-State Conduction in Radial Geometries:

Pipes, pressure vessels and annular fins are engineering examples of radial systems. The governing equation for steady-state one-dimensional conduction in a radial system is

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0 \qquad (2.19)$$

From integrating twice:

$T = C_1 \ln(x) + C_2$, and the constants are determined from the boundary conditions, e.g. if T = T1 at r = r1 and T = T2 at r = r2, then:

$$\frac{T - T_1}{T_2 - T_1} = \left\{ \frac{\ln(r/r_1)}{\ln(r_2/r_1)} \right\} \qquad (2.20)$$

Similarly since the heat flow $Q = -kA(dT/dr)$, then for a length L (in the axial or 'z' direction) the heat flow can be found from differentiating Equation 2.20.

$$Q = \frac{-2\pi LK(T_2 - T_1)}{\ln(r_2/r_1)} \qquad (2.21)$$

To analyse 1-D radial conduction problems:

Write down equations for the heat flow Q (not the flux, q, as in plane systems, since in a radial system the area is not constant, so q is not constant). For example, the heat flow through a pipe wall with convection on the outside and convection on the inside:

$$Q = 2\pi r_1 L h_{inside}(T_{inside} - T_1)$$

$$Q = 2\pi Lk(T_1 - T_2)/\ln(r_2/r_1)$$

$$Q = 2\pi r_2 L h_{outside}(T_2 - T_{outsied})$$

Rearrange, and add to eliminate T1 and T2 (wall temperatures)

$$Q = \frac{2\pi L(T_{inside} - T_{outside})}{\left(\frac{1}{r_1 h_{inside}}\right) + \left(\frac{\ln(r_2 / r_1)}{k}\right) + \left(\frac{1}{r_2 h_{outside}}\right)} \qquad (2.22)$$

The extension to a composite pipe wall (Region 1 of thermal conductivity k1, Region 2 of thermal conductivity k2 . . . . etc.) is fairly straightforward.

Example 2.2

The Figure below shows a cross section through an insulated heating pipe which is made from steel (k = 45 W / m K) with an inner radius of 150 mm and an outer radius of 155 mm. The pipe is coated with 100 mm thickness of insulation having a thermal conductivity of k = 0.06 W / m K. Air at Ti = 60°C flows through the pipe and the convective heat transfer coefficient from the air to the inside of the pipe has a value of hi = 35 W / m2 K. The outside surface of the pipe is surrounded by air which is at 15°C and the convective heat transfer coefficient on this surface has a value of ho = 10 W / m2 K. Calculate the heat loss through 50 m of this pipe.

Solution

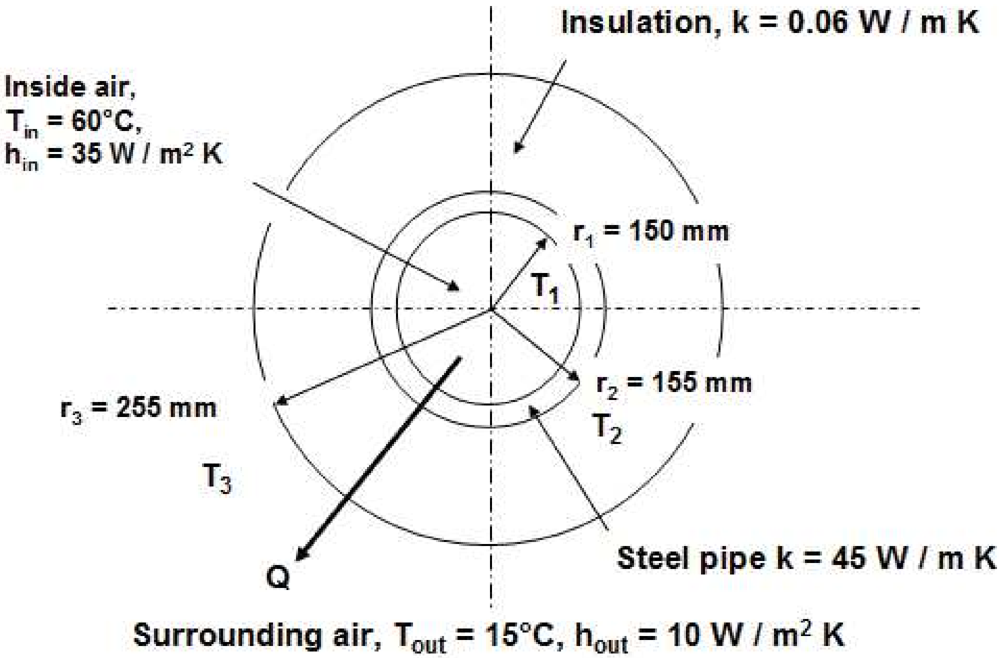


Figure 2-3: Conduction through a radial wall

Unlike the plane wall, the heat flux is not constant (because the area varies with radius). So we write down separate equations for the heat flow, Q.

Convection from inside air to inside of steel pipe

$$Q = 2\pi r_1 L\, h_{in}(T_{in} - T_1)$$

Conduction through steel pipe

$$Q = 2\pi L\, k_{steel}(T_1 - T_2)/\ln(r_2/r_1)$$

Conduction through the insulation

$$Q = 2\pi L\, k_{insulation}(T_2 - T_3)/\ln(r_3/r_2)$$

Convection from outside surface of insulation to the surrounding air

$$Q = 2\pi r_3 L\, h_{out}(T_3 - T_{out})$$

Following the practice established for the plane wall, rewrite in terms of temperatures on the left hand side and then add to eliminate the unknown values of temperature, giving

$$Q = \frac{2\pi L(T_i - T_o)}{\left(\frac{1}{r_1 h_{in}}\right) + \left(\frac{\ln(r_2/r_1)}{k_{steel}}\right) + \left(\frac{\ln(r_3/r_2)}{k_{insulation}}\right) + \left(\frac{1}{r_3 h_{out}}\right)} \qquad (2.23)$$

$$Q = \frac{2\pi \times 50 \times (60-15)}{\left(\frac{1}{35 \times 0.15}\right) + \left(\frac{\ln(0.155/0.150)}{45}\right) + \left(\frac{\ln(0.255/0.155)}{0.06}\right) + \left(\frac{1}{0.255 \times 10}\right)}$$

$$Q = 1592\,Watts$$

Again, the thermal resistance of the insulation is seen to be greater than either the steel or the two resistances due to convection.

Critical Insulation Radius

Adding more insulation to a pipe does not always guarantee a reduction in the heat loss. Adding more insulation also increases the surface area from which heat escapes. If the area increases more than the thermal resistance then the heat loss is increased rather than decreased.

The so-called critical insulation radius is the largest radius at which adding more insulation will create an increase in the heat loss

$$r_{crit} = k_{ins} / k_{ext}$$

Example 2.3

Find the critical insulation radius for the previous question.

Solution:

$$r_{crit} = k_{ins} / h_{ext}$$

$$r_{crit} = 0.06/10$$

$$r_{crit} = 6mm$$

So for r3 > 6 mm, adding more insulation, as intended, will reduce the heat loss.

2.3 Fins and Extended Surfaces

(a) (b)

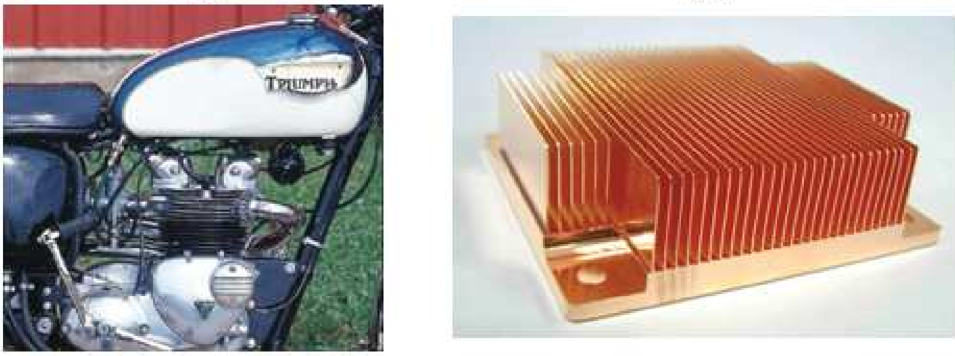


Figure 2-4: Examples of fins (a) motorcycel engine, (b) heat sink

Fins and extended surfaces are used to increase the surface area and therefore enhance the surface heat transfer. Examples are seen on: motorcycle engines, electric motor casings, gearbox casings, electronic heat sinks, transformer casings and fluid heat exchangers. Extended surfaces may also be an unintentional product of design. Look for example at a typical block of holiday apartments in a ski resort, each with a concrete balcony protruding from external the wall. This acts as a fin and draws heat from the inside of each apartment to the outside. The fin model may also be used as a first approximation to analyse heat transfer by conduction from say compressor and turbine blades.

2.3.1 General Fin Equation

The general equation for steady-state heat transfer from an extended surface is derived by considering the heat flows through an elemental cross-section of length δx, surface area δAs and cross-sectional area Ac. Convection occurs at the surface into a fluid where the heat transfer coefficient is h and the temperature Tfluid.

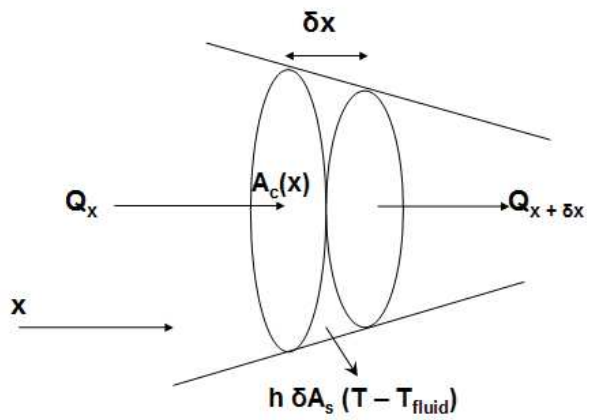


Figure 2-5 Fin Equation: heat balance on an element

Writing down a heat balance in words: heat flow into the element = heat flow out of the element + heat transfer to the surroundings by convection. And in terms of the symbols in Figure 2.5

$$Q_x = Q_{x+\delta x} + h\,\delta A_s (T - T_{fluid}) \qquad (2.24)$$

From Fourier's Law.

$$Q_x = -k\,A_c \frac{dT}{dx} \qquad (2.25)$$

and from a Taylor's series, using Equation 2.25

$$Q_{x+\delta x} = Q_x + \frac{d}{dx}\left(-k\,A_c \frac{dT}{dx}\right)\delta x \qquad (2.26)$$

and so combining Equations 2.24 and 2.26

$$\frac{d}{dx}\left(k\,A_c \frac{dT}{dx}\right)\delta x - h\,\delta A_s (T - T_{fluid}) = 0 \qquad (2.27)$$

The term on the left is identical to the result for a plane wall. The difference here is that the area is not constant with x. So, using the product rule to multiply out the first term on the left hand-side, gives:

$$\frac{d^2T}{dx^2}+\frac{1}{A_c}\frac{dA_c}{dx}\frac{dT}{dx}-\frac{h}{A_ck}\frac{dA_s}{dx}(T-T_{fluid})=0 \qquad (2.28)$$

The simplest geometry to consider is a plane fin where the cross-sectional area, Ac and surface area As are both uniform. Putting $\Theta = T - T_{fluid}$ and letting $m^2 = hp / A_ck$, where P is the perimeter of the cross-section

$$\frac{d^2\Theta}{dx^2}-m^2\Theta=0 \qquad (2.29)$$

The general solution to this is Θ = C1emx + C2e-mx, where the constants C1 and C2, depend on the boundary conditions.

It is useful to look at the following four different physical configurations:
N.B. sinh, cosh and tanh are the so-called hyperbolic sine, cosine and tangent functions defined by:

$$\cosh x=\frac{e^x+e^{-x}}{2},\ \sinh x=\frac{e^x-e^{-x}}{2}\ \text{and}\ \tanh x=\frac{\sinh x}{\cosh x} \qquad (2.30)$$

Convection from the fin tip ($h_{x=L} = h_{tip}$)

$$\frac{T-T_{fluid}}{T_{base}-T_{fluid}}=\frac{\cosh\ m(L-x)+\{(h_{tip}/mk)\sinh\ m(L-x)\}}{\cosh\ (mL)+\{(h_{tip}/mk)\sinh\ (mL)\}} \qquad (2.31)$$

where $h_{base} = h_{x=0}$.

$$Q=(hPkA_c)^{1/2}(T_{base}-T_{fluid})\frac{\sinh\ (mL)+\{(h_{tip}/mk)\cosh\ (mL)\}}{\cosh\ (mL)+\{(h_{tip}/mk)\sinh\ (mL)\}} \qquad (2.32)$$

Adiabatic Tip (htip = 0 in Equation 2.32)

$$\frac{T-T_{fluid}}{T_{base}-T_{fluid}}=\frac{\cosh\ m(L-x)}{\cosh\ (mL)} \qquad (2.33)$$

$$Q = (h\,P\,k\,A_c)^{1/2}\,(T_{base} - T_{fluid})\ \tanh\,(mL) \qquad (2.34)$$

3. Tip at a specified temperature (Tx = L)

$$\frac{T - T_{fluid}}{T_{base} - T_{fluid}} = \frac{\left\{\left(\dfrac{T_{x=L} - T_{fluid}}{T_{base} - T_{fluid}}\right)\ \sinh\,(mx)\right\} + \ \sinh\ m(L - x)}{\sinh\,(mL)} \qquad (2.35)$$

$$Q = (h\,P\,k\,A_c)^{1/2}\,(T_{base} - T_{fluid})\,\frac{\left\{\cosh\,(mL) - \left(\dfrac{T_{x=L} - T_{fluid}}{T_{base} - T_{fluid}}\right)\right\}}{\sinh\,(mL)} \qquad (2.36)$$

Infinite Fin $(T = T_{fluid}\ at\ \ x = \infty)$

$$\frac{T - T_{fluid}}{T_{base} - T_{fluid}} = e^{-mx} \qquad (2.37)$$

$$Q = (h\,P\,k\,A_c)^{1/2}\,(T_{base} - T_{fluid}) \qquad (2.38)$$

2.3.2 Fin Performance

The performance of a fin is characterised by the fin effectiveness and the fin efficiency

Fin effectiveness, ε_{fin}

$\varepsilon_{fin} =$ fin heat transfer rate / heat transfer rate that would occur in the absence of the fin

$$\varepsilon_{fin} = Q / hA_c (T_{base} - T_{fluid}) \qquad (2.39)$$

which for an infinite fin becomes, Q given by

$$\varepsilon_{fin} = (Pk / hA_c)^{1/2} \qquad (2.40)$$

Fin efficiency, η_{fin}

$\eta_{fin} =$ actual heat transfer through the fin / heat transfer that would occur if the entire fin were at the base temperature.

$$\eta_{fin} = Q / hA_s (T_{base} - T_{fluid}) \qquad (2.41)$$

which for an infinite fin becomes, with Q given by Equation 2.38

$$\eta_{fin} = (PkA_c / hA_s^2)^{1/2} \qquad (2.42)$$

Example 2.3

The design of a single 'pin fin' which is to be used in an array of identical pin fins on an electronics heat sink is shown in Figure 2.6. The fin is made from cast aluminium, k = 180 W / m K, the diameter is 3 mm and the length 15 mm. There is a heat transfer coefficient of 30 W / m2 K between the surface of the fin and surrounding air which is at 25°C.

1. Use the expression for a fin with an adiabatic tip to calculate the heat flow through a single pin fin when the base has a temperature of 55°C.

2. Calculate also the efficiency and the effectiveness of this fin design.

3. How long would this fin have to be to be considered "infinite" ?

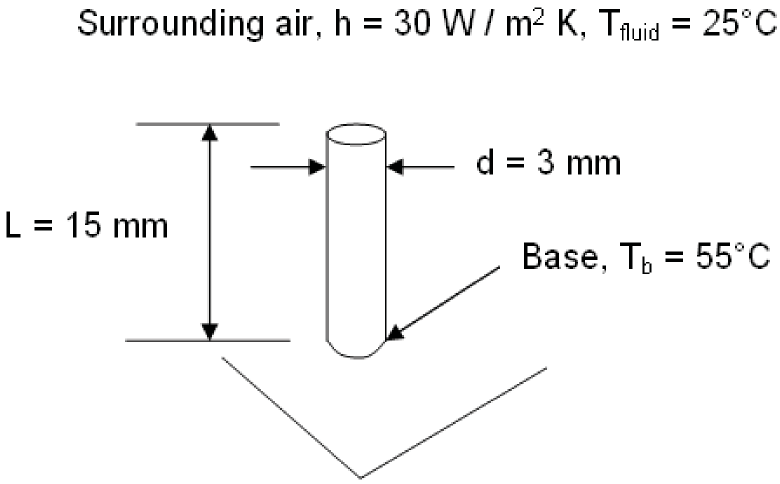


Figure 2-6 Pin Fin design

Solution

For a fin with an adiabatic tip

$$Q = (h\,P\,k\,A_c)^{1/2}\,(T_{base} - T_{fluid})\ \tanh\,(mL)$$

For the above geometry

$$P = \pi\,d\ = 9.42 \times 10^{-3} m$$

$$A_c = \pi\,d^2\ / 4 = 7.07 \times 10^{-6} m^2$$

$$mL = (hP / k\,A_c)^{1/2}\,L = (30 \times 9.42 \times 10^{-3} / 180 \times 7.07 \times 10^{-6}) \times 0.015 = 0.224$$

$$\tanh(mL) = 0.22$$

$$Q = (30 \times 9.42 \times 10^{-3} \times 180 \times 7.07 \times 10^{-6})^{1/2} \times (55 - 25) \times 0.22$$

$$Q = 0.125\,Watts$$

Fin efficiency

$$\eta_{fin} = Q / hA_s (T_{base} - T_{fluid})$$

$$A_s = \pi\,dL = 0.141 \times 10^{-3} m^2$$

$$\eta_{fin} = 0.125 / 30 \times 0.141 \times 10^{-3} \times (55 - 25)$$

$$\eta_{fin} = 0.985\ (98.5\%)$$

Fin effectiveness

$$\varepsilon_{fin} = Q / hA_c(T_{base} - T_{fluid})$$
$$\varepsilon_{fin} = 0.125 / 30 \times 0.707 \times 10^{-6} (55 - 25)$$

$$\varepsilon_{fin} = 19.6$$

For an infinite fin, Tx = L = Tfluid. However, the fin could be considered infinite if the temperature at the tip approaches that of the fluid. If we, for argument sake, limit the temperature difference between fin tip and fluid to 5% of the temperature difference between fin base and fluid, then:

$$\frac{T_{x=L} - T_{fluid}}{T_b - T_{fluid}} = 0.05$$

Using equation 2.33 for the temperature distribution and substituting x = L, noting that cosh (0) = 1, implies that 1/ cosh (mL) = 0.05. So, mL = 3.7, which requires that L > 247 mm.

Simple Time Dependent Conduction

The 1-D time-dependent conduction equation is given by Equation 2.10 with no variation in the y or z directions:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (2.43)$$

A full analytical solution to the 1-D conduction equation is relatively complex and requires finding the roots of a transcendental equation and summing an infinite series (the series converges rapidly so usually it is adequate to consider half a dozen terms). There are two alternative possible ways in which a transient conduction analysis may be simplified, depending on the value of the Biot number (Bi = h L /k).

2.3.3 Small Biot Number (Bi << 1): Lumped Mass Approximation

A small value of Bi implies either that the convective resistance 1/h, is large, or the conductive resistance L/k is small. Either of these conditions tends to reduce the temperature gradient in the material. For example there will be very little difference between the two surface temperatures of a heated copper plate (k ≈ 400 W/m K) of say 5 or 10 mm thickness. Whereas for Perspex (k ≈ 0.2 W/m K), there could be a significant difference. The copper thus behaves as a "lumped mass". Hence for the purpose of analysis we may treat it as a body with a homogenous temperature. A simple heat balance on a material of mass, m, density ρ, specific heat C, exchanging heat by convection from an area A to surrounds at T∞, gives

$$Q = m\,C\frac{d}{dt}(T_s - T_\infty) = -h\,A(T_s - T_\infty) \qquad (2.44)$$

define: $\Theta = (T_s - T_\infty)/(T_{s,initial} - T_{\infty,initial})$ and : $\lambda = A/m\,C$

in forced convection when $h \neq f(T_s - T_\infty)$. This gives the simple solution:

$$\ln\Theta = -\lambda\,h\,t \quad or \quad \Theta = \exp(-\lambda\,h\,t) \qquad (2.45)$$

In free convection when the heat transfer coefficient depends on the surface to fluid temperature difference, say $h \propto (T_s - T_\infty)^n$, then the solution becomes:

$$\Theta^{-n} = 1 + (n\,h_{initial}\,\lambda)\,t \qquad (2.46)$$

2.3.4 Large Biot Number (Bi >>1): Semi - Infinite Approximation

When Bi is large (Bi >> 1) there are, as explained above, large temperature variations within the material. For short time periods from the beginning of the transient (or to be more precise for Fo << 1), the boundary away from the surface is unaffected by what is happening at the surface and mathematically may be treated as if it is at infinity.

There are three so called semi-infinite solutions:

N.B. erfc(x) = 1 - erf(x); erf(x) = error function
Given by the series:

$$erf\ (x) = \frac{2}{\sqrt{\pi}}\left\{x - \frac{x^3}{3} + \frac{1}{2!}\frac{x^5}{5} - \frac{1}{3!}\frac{x^7}{7} + ...\right\} \qquad (2.47)$$

Constant surface heat flux

$$T(x,t) - T_{initial} = \left[\left\{\frac{q}{k}\left(\frac{4\alpha t}{\pi}\right)^{1/2}\right\}\exp\left(\frac{-x^2}{4\alpha t}\right)\right] - \left[\left(\frac{qx}{k}\right) erfc\left\{\frac{x}{(4\alpha t)^{1/2}}\right\}\right] \qquad (2.48)$$

Constant surface temperature

$$\frac{(T(x,t) - T_s)}{(T_{initial} - T_s)} = erf\left(\frac{x}{(4\alpha t)^{1/2}}\right) \qquad (2.49)$$

Constant surface heat transfer coefficient

$$\frac{(T(x,t) - T_s)}{(T_{initial} - T_s)} = erfc\left\{\frac{x}{(4\alpha t)^{1/2}}\right\} - \left[\exp\left(\frac{hx}{k} + \frac{h^2\alpha t}{k^2}\right) erfc\left\{\frac{x}{(4\alpha t)^{1/2}} + \frac{h}{k}(\alpha t)^{1/2}\right\}\right] \qquad (2.50)$$

As well as being useful in determining the temperature of a body at time, these low Biot number and large Biot number methods can also be used in the inverse mode. This is the reverse of the above and makes use of the temperature time history to determine the heat transfer coefficient.

Example 2.4

A titanium alloy blade from an axial compressor for which k = 25 W / m K, ρ = 4500 kg / m3 and C = 520 J/kg K, is initially at 40ºC. Although the blade thickness (from pressure to suction side) varies along the blade, the effective length scale for conduction may be taken as 3mm. When exposed to a hot gas stream at 350ºC, the blade experiences a heat transfer coefficient of 150 W / m2 K. Use the lumped mass approximation to estimate the blade temperature after 50 s.

Firstly, check that Bi << 1
Bi = h L / k = 150 x 0.003 / 25 = 0.018. So the lumped mass method can be used.

Solution

$$\Theta = \exp(-\lambda h t)$$

$$\frac{T - T_{fluid}}{T_{initial} - T_{fluid}} = \exp\left(-\frac{h\,A}{m\,C}t\right)$$

However, the mass m, and surface area A, are not known. It is easy to rephrase the above relationship, since mass = density x volume and volume = area x thickness, where this thickness is the conduction length scale, L. So

$$\frac{T - T_{fluid}}{T_{initial} - T_{fluid}} = \exp(-\frac{h}{\rho C\ L}t)$$

From which

$$T = \left\{ \left(T_{initial} - T_{fluid} \right) \left(\exp(-\frac{h}{\rho\, C\ L} t) \right) \right\} + T_{fluid}$$

$$T = \left\{ \left(40 - 350 \right) \left(\exp(-\frac{150}{4500\, x\ 520\, x\, 0.003} x\, 50) \right) \right\} + 350$$

$$T = 243.5^{\circ} C$$

2.4 Summary

This chapter has introduced the mechanism of heat transfer known as conduction. In the context of engineering applications, this is more likely to be representative of the behaviour in solids than fluids. Conduction phenomena may be treated as either time-dependent or steady state.

It is relatively easy to derive and apply simple analytical solutions for one-dimensional steady-state conduction in both Cartesian (plates and walls) and cylindrical (pipes and pressure vessels) coordinates. Two-dimensional steady-state solutions are much more complex to derive and apply, so they are considered beyond the scope of this introductory level text.

Fins and extended surfaces are an important engineering application of a one-dimensional conduction analysis. The design engineer will be concerned with calculating the heart flow through the fin, the fin efficiency and effectiveness. A number of relatively simple relations were presented for fins where the surface and cross sectional areas are constant.

Time-dependent conduction has been simplified to the two extreme cases of Bi << 1 and Bi >> 1. For the former, the lumped mass method may be used and in the latter the semi-infinite method. It is worth noting that in both cases these methods are used in practical applications in the inverse mode to measure heat transfer coefficients from a know temperature-time history.

In many cases, the boundary conditions to a conduction analysis are provided in terms of the convective heat transfer coefficient. In this chapter a value has usually been ascribed to this, without explaining how and from where it was obtained. This will be the topic of the next chapter.

2.5 Multiple Choice Assessment

2.5.1 Simple 1-D Conduction

1. Which of these statements is a correct expression of Fourier's Law

$$a)\, q = m\, C_p\, \Delta T;\ b)\, q_x = -k \frac{dT}{dx};\ c)\, q_x = -k \frac{\partial T}{\partial y};\ d)\, Q_x = -k \frac{\partial T}{\partial x};\ e)\, Q_x = -k \frac{\partial T}{\partial y}$$

2. Which is the correct form of the 2D steady state conduction equation for constant thermal

$$a)\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t};b)\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0;c)\frac{\partial^2 T}{\partial x^2}+\frac{\partial T}{\partial x}=0;$$

$$d)\frac{d^2 T}{dx^2}+\frac{d^2 T}{dy^2}=0;\ e)\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}=0$$

3. conductivity, in Cartesian coordinates ?
   - Which of the following is NOT a boundary condition ?
   - Tx=L = 50ºC
   - qx=L = qinput
   - –k(dT/dx)x=L = h(Ts – Tf)
   - Ty=L/2 = T0(1 – x/L)
   - k = 16 W/m K

4. The statement Tx=0 = T0, means that:
   - the temperature at x = 0 is zero
   - the temperature at x = 0 is constant
   - the temperature at x = L is zero
   - the temperature at x = L is constant
   - the surface at x = 0 is adiabatic

5. The statement –k(dT/dx)x=L = h(Ts – Tf) means that:
   - the temperature at x = L is constant
   - the heat flux at x = L is constant
   - heat transfer by convection is zero at x = L
   - heat transfer by conduction is zero at x = L
   - heat transfer by convection equals that by conduction at x = L

6. If Bi << 1, then:
   - temperature variations in a solid are significant
   - temperature variations in a solid are insignificant
   - surface temperature is virtually equal to the fluid temperature
   - surface temperature is much less than the fluid temperature
   - surface temperature is much greater than the fluid temperature

7. A large value of heat transfer coefficient is equivalent to:
   - a large thermal resistance
   - a small thermal resistance
   - infinite thermal resistance
   - zero thermal resistance
   - it depends on the fluid temperature

8. A wall 0.1m thick is made of brick with k = 0.5 W/m K. The air adjacent to one side has a temperature of 30ºC, and that on the other 0ºC. Calculate the heat flux through the wall if there is a heat transfer coefficient of 20 W/m2K acting on both sides.
   - 600 W/m2
   - 120 W/m2
   - 150 W/m2
   - 100 W
   - 100 W/m2

9. Which provides the highest thermal resistance in Question 8 ?
   - the conduction path
   - the two convection coefficients
   - conduction and convection give equal thermal resistance
   - there is zero thermal resistance
   - the thermal resistance is infinite

10. What is the appropriate form of the conduction equation for steady-state radial conduction in a pipe wall ?

$$a)\ \frac{\partial^2 T}{\partial r^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t};\ b)\ \frac{\partial^2 T}{\partial r^2} = 0;\ c)\ \frac{\partial^2 T}{\partial x^2} = 0;\ d)\ Q = h\,A\,\Delta T;\ e)\ \frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

11. The temperature, T at radius r within a pipe of inner radius rI and outer radius r0, where the temperatures are TI and T0, respectively, is given by:

$$a)\frac{T-T_i}{T_o-T_i}=\frac{r/r_i}{r_o/r_i};\ b)\frac{T-T_i}{T_o-T_i}=\frac{\log_e r/r_i}{\log_e r_o/r_i};\ c)\frac{T-T_i}{T_o-T_i}=\frac{\log_e r/r_i}{r_o/r_i};$$

$$d)\frac{T-T_i}{T_o-T_i}=\frac{r/r_i}{\log_e r_o/r_i};\ e)\frac{T-T_i}{T_o-T_i}=1-e^{-r/r_o}$$

12. The British Thermal Unit (Btu) is a measure of energy in the British or Imperial system of units. Given that, you should be able to deduce the correct units for thermal conductivity in the Imperial system. What is it ?
- Btu / ft ºF
- Btu / ft hr ºF
- Btu / ft2 hr ºF
- Btu / hr ºF
- Btu

13. Calculate the heat flow through a 100m length of stainless steel (k = 16 W/ m K) pipe of 12 mm outer diameter and 8 mm inner diameter when the surface temperature is 100ºC on the inside and 99.9ºC on the outside.
- 800 W
- 670 W
- 3 kW
- 2.5 kW
- 2 kW

14. Applied to a pipe , the critical insulation radius describes a condition when:
- the flow is turbulent
- the heat flow is infinite
- the heat flow is a maximum
- the heat flow is a minimum
- the heat flow is zero

15. The (approximate) value of thermal conductivity of pure copper is:
- 1 W / m K
- 40 W / m2 K
- 40 W / m K
- 400 W / m2 K
- 400 W / m K

16. Which of the following statements is true ?
    - mild steel is a better conductor than stainless steel
    - mild steel has lower thermal conductivity than stainless steel
    - for the same thickness, mild steel has greater thermal resistance than stainless steel
    - both mild steel and stainless steel are better thermal conductors than aluminium
    - mild steel and stainless steel have more or less the same thermal conductivity

17. Values of thermal conductivity for many engineering materials (solids, liquids and gases) can be found in:
    - the steam tables
    - the Guardian
    - Kaye and Laby, Tables of Physical Constants
    - lecture notes
    - tabulated data at the back of a good heat transfer textbook

18. For 1-D conduction in a plane wall, the temperature distribution is:
    - parabolic
    - logarithmic
    - linear
    - quadratic
    - trigonometric

19. A good insulator has:
    - a large value of k
    - a small value of k
    - an infinite value of k
    - a large value of h
    - a large value of h and a small vale of k

20. Which of the following statements is true ?
    - In gases, k increases with increasing temperature, whereas in metals, k decreases with temperature
    - In both gases and metals, k decreases with increasing temperature
    - In both gases and metals, k increases with increasing temperature
    - In both gases and metals, k is more or less independent of temperature
    - In gases, k decreases with increasing temperature, whereas in metals k increases with temperature

2.5.2 Fins and Time Dependent Conduction

1. Which of the following is NOT an example of a fin ?
   - ribs on an electric motor casing
   - a concrete balcony protruding from a wall
   - a turbine blade in a hot gas path
   - an insulated pipe carrying high pressure steam
   - porcupine spines

2. Which is NOT a boundary condition for a fin analysis ?
   - $T \rightarrow T_f$ as $L \rightarrow \infty$
   - $(dT/dx)_{x=L} = 0$
   - $-k(dT/dx)_{x=0} = h_{tip}(T_{x=L} - T_f)$
   - hP/kA_c = constant
   - $T_{x=L}$ = constant

3. A circular fin of 5 mm diameter has a length of 30 mm, what is its perimeter, P ?
   - 70 mm
   - 19.6 mm
   - 15.7 mm
   - 60 mm
   - 7.85 mm

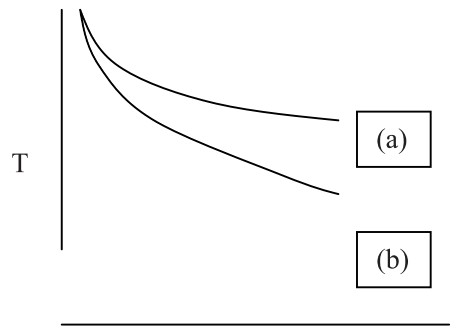
4. What is the cross sectional area, Ac, in the above example (Q. 3) ?
   - 471 mm2
   - 15.7 mm2
   - 490 mm2
   - 19.6 mm2
   - none of these

5. The fin parameter, m, is defined as:
   - m = (h / k Ac)1/2
   - m = (h P / k Ac)1/2
   - m = (h P/ k Ac)
   - m = (h P/ k)
   - m = (h L/ k)1/2

6. The equation for the temperature distribution in a fin of length L and with an adiabatic tip is given by which of the following ?
   - (T – Tf) = (Tb – Tf) cosh m (L – x) / cosh mL
   - (T – Tf) = (Tb – Tf) e - mx
   - (T – Tf) = (Tb – Tf) (h P k Ac)1/2 tanh mL
   - (T – Tf) = (Tb – Tf) (h P k Ac)1/2
   - none of these

7. Cosh(x) = ?
   - cos(x) because the 'h' is a typographical error
   - ex + e-x
   - (ex + e-x) / 2
   - ex - e-x
   - (ex - e-x) / 2

8. The diagram below shows temperature distributions along the length of two geometrically identical fins, experiencing the same convective heat transfer coefficient but made from different materials. Which material, a or b, has the higher value of thermal conductivity ?



- x
- material (a)
- material (b)
- both (a) and (b) have the same thermal conductivity
- the temperature distribution is independent of thermal conductivity
- it's not that simple

9. Fin efficiency is defined as:
   - tanh (mL)
   - (h P / k Ac)1/2
   - (heat transfer with fin) / (heat transfer without fin)
   - (actual heat transfer through fin) / (heat transfer assuming all fin is at T = Tb)
   - (Tx=L – Tf) / (Tb – Tf)

10. For an infinite fin, the temperature distribution is given by: (T – Tf) / (Tb – Tf) = e – mx. The heat flow through the fin is therefore given by:
    - k (Tb – Tf) / L
    - zero, because the fin is infinite
    - infinite because the fin is infinite
    - (Tb – Tf) (h P / k Ac)1/2
    - (Tb – Tf) (h P / k Ac)1/2 tanh (mL)

11. The Biot number, Bi, is defined as:
    - Bi = h k / L
    - Bi = h L / k
    - Bi = k / L H
    - Bi = q L / k
    - Bi = ρ U L / k

12. For a plate of length L, thickness, t, and width, W, subjected to convection on the two faces of area L x W. What is the correct length scale for use in the Biot number ?
    - L
    - W
    - t
    - t / 2
    - L / 2

13. If Bi << 1, then this implies:
    - heat transfer is negligible
    - the surface is insulated
    - conduction is time dependent
    - temperature variations within the solid are negligible
    - temperature variations within the solid are significant

14. The units of thermal diffusivity, α, are:
    - kg / m s
    - m / s
    - m2 / s
    - m / s2
    - kg / m2 s

15. The fourier number, Fo is defined as:
    - Fo = α k / L2
    - Fo = α t / L2
    - Fo = ρ k / L2
    - Fo = k t / L2
    - Fo = L2 / α t

16. If Fo >> 1, then this implies
    - a steady state flow
    - steady state conduction
    - time dependent flow
    - time dependent conduction
    - 2-D conduction

17. Under what circumstances can the lumped mass method be used ?
    - Bi >> 1; b) Bi << 1; c) Bi = 1; d) Fo >> 1; e) when the object is a small lump of mass

18. Under what circumstances can the semi-infinite approximation be used ?
    - Bi >> 1; b) Bi << 1; c) Bi = 1; d) Fo >> 1; e) when the object is at least 2m thick

19. The temperature variation with time using the lumped mass method is given by:
    - (T – Tf) / (Tinitial – Tf) = e –λ t, what is λ ?
    - h A / m k
    - h A / m L
    - h A / m C
    - h α / m C
    - α / C

20. In free convection, the heat transfer coefficient depends on the temperature difference between surface and fluid. Which of the following statements is true ?
    - the lumped mass equation given in Q.19 may be used without modification
    - it is possible to use the lumped mass method, but with modification
    - it is not possible to use the lumped mass method at all
    - only the semi-infinite method may be used
    - need to use CFD

3. Convection

In Chapter 1, we introduced the three modes of heat transfer as conduction, convection and radiation. We have analysed conduction in Chapters 2 in more detail and we only used convection to provide a possible boundary condition for the conduction problem.

We also described convection briefly in Chapter 1 as the energy transfer between a surface and a fluid moving over the surface. Although the mechanism of molecular diffusion contributes to convection, the dominant mechanism is the bulk motion of the fluid particles.

We have also found that the conduction is dependent on the temperature gradient and thermal conductivity which is a physical property of the material. On the other hand, convection is a function of the temperature difference between the surface and the fluid and the heat transfer coefficient. The heat transfer coefficient is not a physical property of the material, but it rather depends on a number of parameters including fluid properties as well as the nature of the fluid motion.

Thus to obtain an accurate measure of the convective heat transfer coefficient requires analysis of the flow pattern in the vicinity of the surface in concern. The nature of the flow motion will depend on the geometry and boundary conditions to the region of interest.

Consequently, in this chapter we will develop basic methods used to characterise the flow leading to the calculation of the convective heat transfer coefficient. The concept of boundary layer will be introduced and distinction will be made between laminar and turbulent boundary layers and also the concept of transition from laminar to turbulent boundary layer will be discussed.

We will also distinguish two types of flow motion leading to two distinct mechanisms of heat transfer by convection. Forced convection, where the flow is pushed against a surface by external means, such as blowing, and natural convection where flow motion is due to the action of density variations leading to flow motion caused by body forces.

It is apparent from the above that convection is a complex physical phenomena governed by a large number of parameters. One way of allowing a systematic theoretical analysis is the use of the concept of dimensional analysis, which reduces the number of controlling parameters to few non-dimensional groupings. These lead to more general formulations for the convective heat transfer coefficient.

3.1 The convection equation

Heat conducted through a solid must eventually find its way to the surroundings. In most cases the heat supplied to this solid comes also from its surroundings i.e. heat transferred from the air of a room to the room walls then through the walls and eventually out into the ambient air.

This led to the introduction of Newton's equation.

$$Q = A\,h\;(T_s - T_\infty) \qquad [W] \qquad (3.1)$$

T_s : temperature of the wall surface

T_∞ : temperature of the fluid away from the wall

A : heat transfer area

Ts, T∞ and A are measurable quantities. Evaluation of the convective coefficient h then completes the parameters necessary for heat transfer calculations. The convective coefficient h is evaluated in some limited cases by mathematical analytical methods and in most cases by experiments.

3.2 Flow equations and boundary layer

In this study, we are going to assume that the student is familiar with the flow governing equations. The interested reader can refer to basic fluid dynamics textbooks for their derivation. However, for clarity, we are going to state the flow equations to provide a more clear picture for the boundary layer simplifications mentioned below.

Fluid flow is covered by laws of conservation of mass, momentum and energy. The conservation of mass is known as the continuity equation. The conservation of momentum is described by the Navier-Stokes equations while the conservation of energy is described by the energy equation. The flow equations for two dimensional steady incompressible flow in a Cartesian coordinate system are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{(Continuity)} \qquad (3.2a)$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + F_x \qquad \text{(x-momentum)} \qquad (3.2b)$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + F_y \qquad \text{(y-momentum)} \qquad (3.2c)$$

$$\rho\, c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\Phi \qquad \text{(Energy)} \qquad (3.2d)$$

where u and v are the flow velocities in the x and y directions, T is the temperature, p is the pressure, ρ , μ and c_p are the fluid density, viscosity and specific heat at constant pressure, F_x and F_y are the body forces in the x and y directions and Φ is the dissipation function.

3.2.1 The velocity boundary layer

Figure 3.1 shows fluid at uniform velocity U_∞ approaching a plate and the resulting development of the velocity boundary layer. When the fluid particles make contact with the surface they assume zero velocity. These particles then tend to retard the motion of particles in the fluid layer above them which in term retard the motion of particles above them and so on until at a distance $y = \delta$ from the plate this effect becomes negligible and the velocity of the fluid particles is again almost equal to the original free stream velocity U_∞ .

The retardation of the fluid motion which results in the boundary layer at the fluid-solid interface is a result of shear stresses (τ) acting in planes that are parallel to the fluid velocity. The shear stress is proportional to the velocity gradient and is given by

$$\tau = \mu \frac{du}{dy} \qquad (3.3)$$

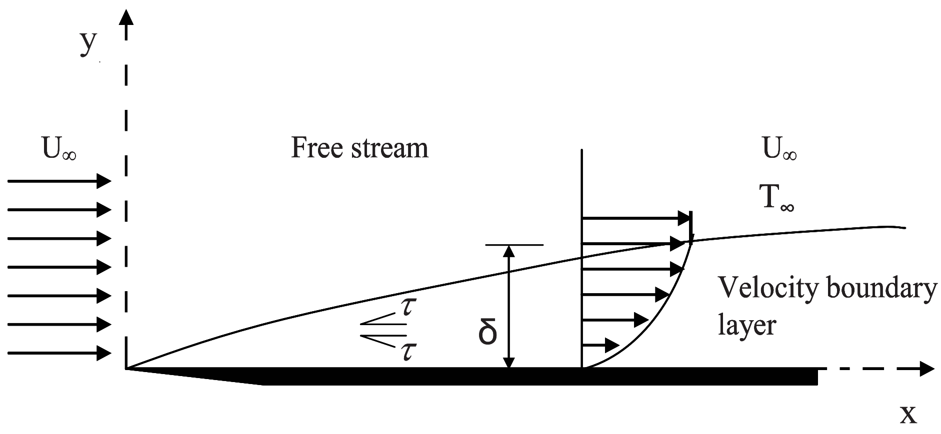


Figure 0-1The velocity boundary layer on a flat plate

The fluid flow as described above is characterized by two distinct regions: a thin fluid layer – the boundary layer – in which velocity gradient and shear stresses are large and an outer region – the free stream – where velocity gradients and shear stresses are negligible.

The quantity δ seen in the above figure is called the boundary layer thickness. It is formally defined as the value of y at which

$$U = 0.99\,U_\infty \qquad (3.4)$$

The boundary layer velocity profile refers to the way in which the velocity u varies with distance y from the wall through the boundary layer.

3.2.2 Laminar and turbulent boundary layer

In convection problems it is essential to determine whether the boundary layer is laminar or turbulent. The convective coefficient h will depend strongly on which of these conditions exists.

There are sharp differences between laminar and turbulent flow conditions. In laminar boundary layers the fluid motion is highly ordered. Fluid particles move along streamlines. In contrast, fluid motion in the turbulent boundary layer is highly irregular. The velocity fluctuations that exist in this regular form of fluid flow result in mixing of the flow and as a consequence enhance the convective coefficient significantly.

Figure 3.2 shows the flow over a flat plate where the boundary layer is initially laminar. At some distance from the leading edge fluid fluctuations begin to develop. This is the transition region. Eventually with increasing distance from the leading edge complete transition to turbulence occurs. This is followed by a significant increase in the boundary layer thickness and the convective coefficient, see Figures 3.2 and 3.4. Three different regions can be seen in the turbulent boundary layer. The laminar sublayer, the buffer layer and turbulent zone where mixing dominates. The location where transition to turbulence exists is determined by the value of theReynolds's number which is a dimensionless grouping of variables.

$$\mathrm{Re} = \frac{\rho U_\infty \mathrm{L}}{\mu} \qquad (3.5a)$$

Where L is the appropriate length scale.

In the case of the flat plate the length is the distance x from the leading edge of the plate. Therefore,

$$\mathrm{Re}_x = \frac{\rho U_\infty x}{\mu} \qquad (3.5b)$$

For Rex < 5×105 the flow is laminar and for Rex > 5×105 the flow is turbulent

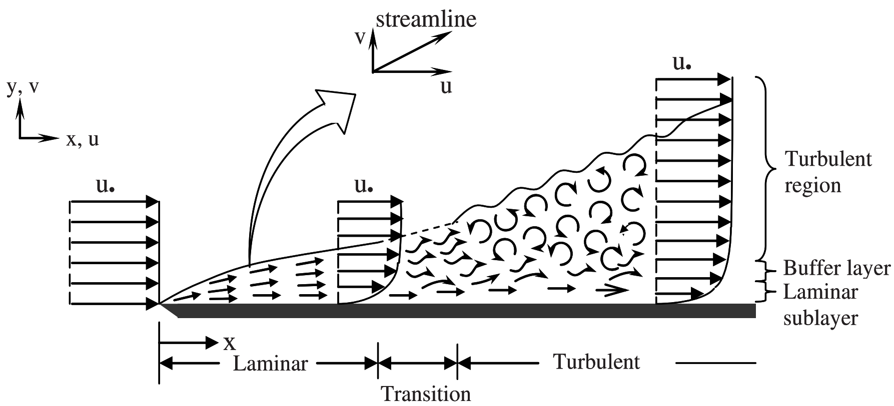


Figure 3.2 Laminar and turbulent boundary layer on a flat plate

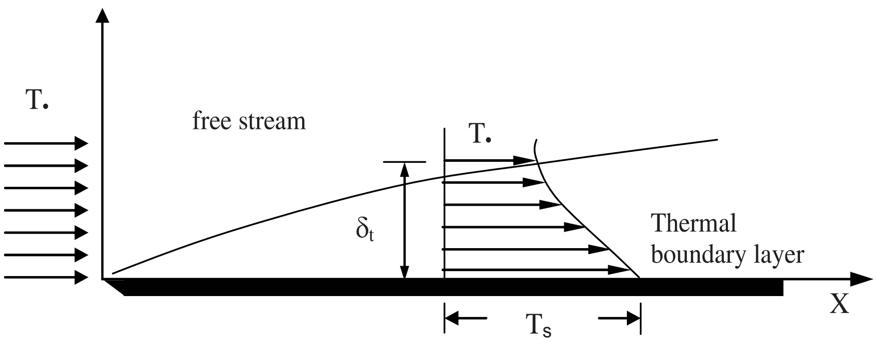


Figure 3.3The thermal boundary layer development on a flat plate.

3.2.3 The Thermal boundary layer

Figure 3.3 shows analogous development of a thermal boundary layer. A thermal boundary layer must develop similar to the velocity boundary layer if there is a difference between the fluid free stream temperature and the temperature of the plate. The fluid particles that come in contact with the plate achieve thermal equilibrium with the surface and exchange energy with the particles above them. A temperature gradient is therefore established.

The quantity δ_t is the thickness (at any position x) of the thermal boundary layer and is formally defined as the value of y for

$$\frac{T_s - T}{T_s - T_\infty} = 0.99 \tag{3.6}$$

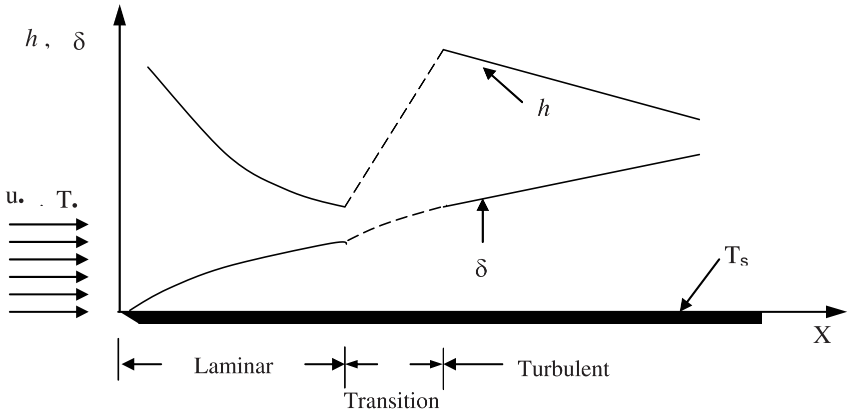


Figure 3.4 Variation of the velocity boundary layer thickness and the local convective heat transfer coefficient

At the surface of the plate and at any distance x the heat flux is given

$$q_x = -k\frac{dT}{dy}$$

where $\frac{dT}{dy}$ is evaluated at the wall-fluid interface and k is the conductivity of the fluid at the wall temperature. Fourier's law above can be applied because at the interface the fluid is at rest and conduction occurs.

Then

$$q_x = -k\frac{dT}{dy} = h_x\,(T_s - T_\infty) \tag{3.7}$$

Therefore

$$h_x = -\frac{1}{T_s - T_\infty}k\frac{dT}{dy} \tag{3.8}$$

Because δt increases with x, the temperature gradient decrease with distance x (since the thermal boundary layer thickness increases) and therefore hx decreases with distance.

Equation 3.8 indicates that to be able to calculate the heat transfer coefficient, we need to know the temperature gradient in the boundary layer. To achieve this, either accurate measurements of the temperature distribution normal to the wall are required, or the flow equations (Equations 3.2) need to be solved to obtain the temperature distribution.

Analytic solutions to the flow equations are only possible for simple geometries and simple flows with various further simplifications. For most problems encountered in engineering applications, it is not possible to obtain analytic solutions to those equations. We are going to discuss the simplification to the flow equations for laminar boundary layers in the next subsection. Generally, three approaches are possible;

- Measurement of temperature distribution. This is a reliable approach as long as care is taken to obtain accurate measurements. However, this approach has several drawbacks. Measurements of the actual geometries might not be possible at the design stage. The measurements can be costly and time consuming.

- Numerical solution of the flow equation. This is becoming increasingly popular approach with the recent development of Computational Fluid Dynamics (CFD) techniques and the availability of cheap computing power. However, good knowledge of the boundary conditions is required. In addition, the accuracy is limited by the accuracy of the numerical procedure.

- The non-dimensionalisation procedure described in this Chapter is used with the analytical, numerical or experimental approaches mentioned above to reduce the number of analyses required. As we will see in more detail later, the controlling parameters are reduced to a much smaller number of non-dimensional groups, thus making use of the dynamic similarity in the calculation of the heat transfer coefficient.

3.2.4 Flow inside pipes

As seen in Figure 3.5, when fluid with uniform velocity U_∞ enters a pipe, a boundary layer develops on the pipe surface. This development continues down the pipe until the boundary layer is thick enough so it merges at the pipe centreline. The distance it takes to merge is called hydrodynamic length. After that the flow is termed fully developed and is laminar if Red < 2300 (where Red is the Reynolds number based on the pipe diameter, d, and the fluid mean velocity inside the pipe, um, i.e. ($\mathrm{Re}_d = \rho u_m d / \mu$) and turbulent if Red > 4000 with the corresponding velocity profiles as shown in the figure.

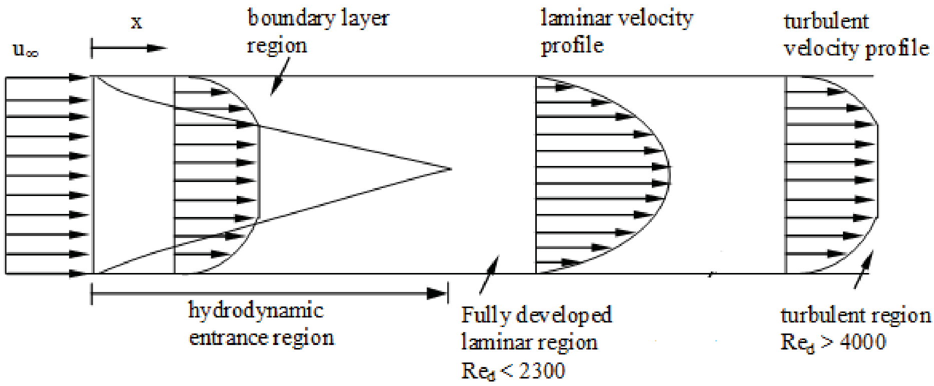


Figure 3-5 Velocity boundary layer development in pipes (not to scale)

If the fluid and the pipe are at different temperatures then a thermal boundary layer develops in the pipe. This is shown in Figure 3.6 below for the case Ts > Tf. The above description of the hydrodynamic and thermal behaviour of the fluid is important because the convective heat transfer coefficient depends on whether the velocity and thermal fields are developing or developed and whether the flow is laminar or turbulent. This variation is similar to the variation of the convective coefficient with varying velocity and thermal fields on a flat plate see Figure 3.3.

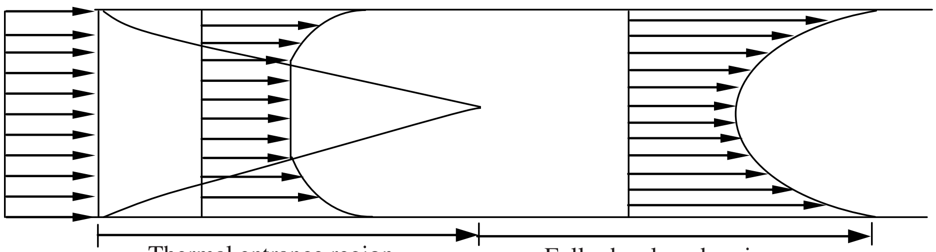


Figure 3.6 Thermal boundary layer development on a heated tube

3.2.5 The boundary layer approximation

Although it is possible to solve the Navier-Stokes equations analytically for simple geometries and simple flows, it is useful to make use of the nature of the boundary layer to simplify the equations before attempting to solve them.

The boundary layer approximation is a simplification which recognises that the flow and temperature distribution in the boundary layers plays the most important role in affecting heat transfer. This leads to the so-called Boundary Layer Equations that are a simplified form of the Navier-Stokes equations. These arise from the simple observation, that the boundary layer thickness is much smaller than the length scale of the geometry in concern. Obviously, this is valid for a wide range of situations but not necessarily all.

Using this simplification, it is possible to neglect certain terms in the flow equations based on an order of magnitude analysis (see Long 1999). For a 2-dimensional flow in the (x,y) plane, the boundary layer equations can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \text{(Continuity)}$$

$$\rho\left\{u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right\}=-\frac{dp}{dx}+\mu\frac{\partial^2 u}{\partial y^2}+F_x \qquad \text{(Momentum)}$$

$$\rho c_p\left\{u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right\}=k\frac{\partial^2 T}{\partial y^2} \qquad \text{(Energy)}$$

These equations assume incompressible flow $(\rho = \text{constant})$ and $\Phi = 0$.

This simplified form can then be solved to obtain the temperature profile near the wall and hence the heat transfer coefficient. Typically, the equations are first integrated term by term. An assumption for the shape of the velocity profile is assumed in the form of a polynomial. The coefficients of the polynomial can then be determined using the boundary conditions. This is then used to obtain the final solution. For the case of laminar flow over a flat plate, see Long (1999).

It is worth mentioning here that despite the major simplifications made by the boundary layer approximation, these equations are still difficult to solve analytically and this is only possible for simple geometries and simple flows. One example is the laminar flow over a flat plate mentioned above. Thus engineers resort to numerical methods based on Computational Fluid Dynamics (CFD) for solution of more complex problems.

3.2.6 Scale analysis: Laminar forced convection for a flat plate

We can further simplify the above equations using scale analysis to obtain a relation between the heat transfer coefficient and flow parameters for a laminar flow over a flat plate. This analysis serves to confirm the validity of the analytic or numerical solutions. From Equation 3.8, the heat transfer coefficient is given by:

$$h = \frac{-k(dT/dy)_{y=0}}{(T_s - T_\infty)}$$

Which becomes:

$$h \sim \frac{k\Delta T}{\delta_t \Delta T} = \frac{k}{\delta_t} \qquad (3.9)$$

where the symbol ~ indicates (of the same order of magnitude).

If we define the heat transfer coefficient in terms of $Nu = hL/k$ a non-dimensional quantity called the Nusselt number, which will be elaborated upon later, then

$$Nu \sim \frac{L}{\delta_t} \qquad (3.10)$$

So if we also take into account that:

$x \sim L$, $y \sim \delta$, $u \sim U_\infty$

Then from the continuity equation, we get:

$$\frac{U_\infty}{L} \sim \frac{v}{\delta}, \quad \text{or} \quad v \sim U_\infty \frac{\delta}{L} \qquad (3.11)$$

The momentum equation in terms of scale becomes:

$$\left\{ \left(\frac{U_\infty^2}{L} \right), v\left(\frac{U_\infty}{\delta} \right) \right\} \sim \left(\frac{\mu}{\rho} \right) \left(\frac{U_\infty}{\delta^2} \right) \qquad (3.12)$$

Substituting the estimate from Equation 3.11:

$$\left\{ \left(\frac{U_\infty^2}{L} \right), \left(\frac{U_\infty^2}{L} \right) \right\} \sim \left(\frac{\mu}{\rho} \right) \left(\frac{U_\infty}{\delta^2} \right) \qquad (3.13)$$

From Equation 3.13, we notice that each of the inertial terms (on the left hand side) of the momentum equation are of comparable magnitude, hence:

$$\left\{ \left(\frac{U_\infty^2}{L} \right) \right\} \sim \left(\frac{\mu}{\rho} \right) \left(\frac{U_\infty}{\delta^2} \right) \qquad (3.14)$$

Rearranging: we get:

$$\frac{\delta}{L} \sim \left(\frac{\mu}{\rho U_{\infty} L}\right)^{\frac{1}{2}} = \mathrm{Re}_L^{-1/2} \qquad (3.15)$$

Exact analysis of the equations shows that:

$$\frac{\delta}{x} = 4.64\,\mathrm{Re}_L^{-1/2} \qquad (3.16)$$

From a similar order of magnitude analysis for the energy equation, we obtain the following relation for oils and gases (but not for fluids with Pr << 1)

$$\frac{\delta_t}{L} \sim \mathrm{Re}_L^{-1/2}\,\mathrm{Pr}^{-1/3} \qquad (3.17)$$

Where Pr is the Prandtl number defined as: $\mathrm{Pr} = \frac{\mu c_p}{k}$

and using Equation 3.10, we obtain:

$$Nu \sim \mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3} \qquad (3.18)$$

Exact analysis leads to the relation:

$$Nu = 0.664\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3} \qquad (3.19)$$

Which is the average heat transfer coefficient for length L of the flat plate.

Although the complete derivation of the analytic solutions was not shown here because they are out of the scope of this introductory book, it is worth mentioning two points. The first is that even for the simplest geometry and the simple laminar flow over a flat plate, the analysis procedure is tedious. However, the order of magnitude analysis proves to be useful tool in revealing the relationship between the Nusselt number and the heat transfer coefficient with the basic flow features and fluid properties.

3.3 Dimensional analysis

Dimensional analysis has its foundations in the simple fact that units on both sides of an equation must be the same. Convective heat transfer is an example of a problem which can be very difficult to solve mathematically. An alternative is then dimensional analysis which attempts to relate important physical quantities i.e. velocity, temperature, fluid properties in dimensionless groups.

There are several methods of reducing a number of dimensional variables into a smaller number of dimensionless groups. The scheme given here was proposed in 1914 by Buckingham, and is called the Buckingham pi theorem. The name pi comes from the mathematical notation π, meaning a product of variables. The dimensionless groups found from the theorem are power products denoted π_1, π_2, π_3, etc. The theorem states that:

"If an equation involving n variables is dimensionally homogenous, it can be reduced to a relationship among n - r independent dimensionless products were r is the minimum of reference dimensions required to describe the variables".

Although the pi theorem is a simple one, its proof is not simple and will not be included here.

The pi theorem is based on the idea of dimensional homogeneity of the equation in question. All theoretically derived equations are dimensionally homogeneous- that is, the dimensions on the left side of the equation must be the same as those in the right side, and all additive separate terms must have the same dimensions.

To be specific, assume that for any physically meaningful equation involving n variables such as:

$$u_1 = f(u_2, u_3, u_4, ..., u_n) \quad (3.20)$$

The dimensions of the variable on the left side of the equation must be equal to the dimensions of any term that stands by itself on the right hand side of the equal sine. It then follows that we can rearrange the equation into a set of dimensionless products so that:

$$\pi_1 = f(\pi_2, \pi_3, \pi_4, ..., \pi_{n-r}) \qquad (3.21)$$

The required number of pi terms is fewer than the number of original variables by r, where r is determined by the minimum number of reference dimensions required to describe the original list of variables. Usually, the basic reference dimensions required to describe the variables will be the basic dimensions M(Mass), L(Length) T(Time) and θ (Temperature). As an alternatives it is possible to use F(Force), L, T, and θ.

The following basic steps can be followed to obtain a set of dimensionless groups from an equation:

List and count the n variables involved in the problem. If any important variables are missing, dimensional analysis will fail.

1. List the dimensions of each variable according to $MLT\theta$, or $FLT\theta$.
2. Find j: initially guess j equal to the number of different dimensions present, and look for j variables which do not form a π product. If no luck, reduce j by 1 and look again. With practice, you will find j rapidly.
3. Select j scaling parameters which do not form a π product.
4. Add a one dimensional variable to your j repeating variables and form a power product. Algebraically find the exponents which make the product dimensionless. Do this sequentially adding a new variable each time.
5. Write the final dimensionless functions, and check your work to make sure all pi groups are dimensionless.

We will use this procedure in the next two subsections to find the non-dimensional groupings for forced and natural convection respectively.

3.3.1 Forced convection

In this case, the heat transfer coefficient h depends on the flow velocity u, characteristic length scale L, fluid conductivity k, viscosity μ, specific heat capacity c_p and density ρ .These parameters and their dimensions are listed in Table 3.1 using the fundamental dimensions of M for mass, L for length, T for time and θ for temperature

Table 0-1: Variables and dimensions for forced convection

Parameter	Dimensions
h	$MT^{-3}\theta^{-1}$
u	LT^{-1}
L	L
k	$MLT^{-3}\theta^{-1}$
μ	$ML^{-1}T^{-1}$
c_p	$L^2T^{-2}\theta^{-1}$
ρ	ML^{-3}

We start then by writing the functional relationship as;

$$h = f(u, L, k, \mu, c_p, \rho) \tag{3.22}$$

The number of variables is 7 and the number of dimensions is four, so we will be able to get three non-dimensional parameters. We choose a set of repeated variables containing the four dimensions such that they do not form a π group on their own. By inspection, we can see that the variables k,L, μ and ρ cannot form a π because only the first one contains the dimension θ .

We then use this set of variables with one of the remaining variables one at a time to extract the non-dimensional groupings.

$$k^a L^b \mu^c \rho^d h = (MLT^{-3}\theta^{-1})^a (L)^b (ML^{-1}T^{-1})^c (ML^{-3})^d (MT^{-3}\theta^{-1}) = M^0 L^0 T^0 \theta^0 \tag{3.23}$$

We can then create a system of linear equations to compute the values of the exponents a,b,c and d that lead to a non-dimensional group. So:

For M: $a + c + d + 1 = 0$

For T: $-3a - c - 3 = 0$

For θ: $-a - 1 = 0$

For L: $a + b - c - 3d = 0$

Solving these equations simultaneously gives:

$a = -1$, $b = 1$, $c = 0$, $d = 0$

This leads to the following non-dimensional group $k^{-1}Lh$. Thus the first non-dimensional group is:

$$\pi_1 = \frac{hL}{k}$$

Repeating the same process with the variable u

$$k^a L^b \mu^c \rho^d u = (MLT^{-3}\theta^{-1})^a (L)^b (ML^{-1}T^{-1})^c (ML^{-3})^d (LT^{-1}) = M^0 L^0 T^0 \theta^0 \tag{3.24}$$

Then:

For M: $a + c + d = 0$
For T: $-3a - c - 1 = 0$
For θ: $-a = 0$
For L: $a + b - c - 3d + 1 = 0$

Solving these equations simultaneously gives:

$a = 0$, $b = 1$, $c = -1$, $d = 1$

This results in the following non-dimensional group:

$$\pi_2 = \frac{\rho u L}{\mu}$$

Repeating the procedure using the variable c_p leads to a third non-dimensional group of the form:

$$\pi_3 = \frac{\mu c_p}{k}$$

Using the above non-dimensional groups, the functional relation in Equation 3.22 can be expressed as:

$$\frac{hL}{k} = f\left(\frac{\rho u L}{\mu}, \frac{\mu c_p}{k}\right) \quad (3.25)$$

It is worth mentioning that the combinations of non-dimensional groupings are not unique and one might get three different sets if the repeated variables used above were different. However, we intentionally chose a set that produces non-dimensional groups which are most widely used in the literature.

We will now give a physical interpretation of the three non-dimensional groups derived above:

$\frac{hL}{k}$: This group is called the Nusselt number Nu. This represents the dimensionless heat transfer coefficient and can be thought of as the ratio of the heat transfer by convection to that of conduction through the fluid. A value of Nusselt number around 1 implies either convective effects are weak or not present. The local Nusselt number is usually termed Nux. If the average value is used over a surface, then the term $\overline{Nu}_L$ is used.

$\frac{\rho u L}{\mu}$: This group is known as the Reynolds number Re. It represents the ratio of the inertia forces to the viscous forces in the fluid. Its value can give an indication of the state of the boundary layer and whether the flow is laminar, turbulent or in transition. Again here the local Reynolds number is termed Rex and the Reynolds number based on the length scale of the flow domain is termed ReL.

$\frac{\mu c_p}{k}$ The Prandtl number Pr. This can be written as follows:

$$\Pr = \frac{\mu c_p}{k} = (\mu / \rho)/(k / \rho c_p) = \nu / \alpha$$

From which, we can se the Pr is the ratio of the momentum diffusivity (ν) to the thermal diffusivity (α). It provides a measure of the relative effectiveness of the transport by diffusion of momentum and energy in the velocity and the thermal boundary layers respectively. For gasses

Pr~1.0 and in this case momentum and energy transfer by diffusion are comparable. In liquid metals Pr << 1 and the energy diffusion rate is much greater than the momentum diffusion rate. For oils Pr >> 1 and the opposite is true.

From the above interpretation it follows that the Pr number affects the growth of the velocity and the thermal boundary layers, i.e. in laminar flow

$$\frac{\text{Velocity boundary layer thickness}}{\text{Thermal boundary layer thickness}} = \text{Pr}^n \,(n \text{ is a positive number})$$

Equation 3.25 forms the basis for general formulations for the non-dimensional heat transfer coefficient as a function of the Reynolds number and the Prandtl numbers as follows:

$$Nu \propto (\text{Re})^a \ (\text{Pr})^b \quad \text{or} \quad Nu = C \ (\text{Re})^a \ (\text{Pr})^b \qquad (3.26)$$

The constants C, a and b can be obtained either by experiments, numerical methods or from analytic solutions if they were possible. In the following subsection we will present the values of these coefficients for common geometries encountered in engineering applications.

It is worth at this juncture to compare Equation 3.26 with Equation 3.19, where the constants C, a and b are given as 0.644, 1/2 and 1/3 for laminar flow over a flat plate.

3.4 Forced Convection relations

In this section, we will present forced convection relations for various geometric situations encountered in engineering applications. These are either analytic or empirical relations (obtained from experiments. These will be supported by worked examples to help the student in understanding the application of these relations.

3.4.1 Laminar flow over a flat plate

In the simple case of an isothermal flat plate, as mentioned earlier, analytical methods can be used to solve either the Navier-Stokes equations or the boundary layer approximation (section 3.2.5) for Nux. This then can be integrated to obtain an overall Nusselt number NuL for the total heat transfer from the plate. The derivation of the analytic solution for the boundary layer equations can be found in Long (1999). This leads to the following solutions:

$$\frac{h_x x}{k} = 0.332 \left(\frac{\mu \, c_p}{k}\right)^{\frac{1}{3}} \left(\frac{\rho \, u_\infty \, x}{\mu}\right)^{\frac{1}{2}} \qquad (3.27)$$

or

$$Nu_x = 0.332 \ (\text{Pr})^{\frac{1}{3}} \ (\text{Re}_x)^{\frac{1}{2}} \qquad (3.28)$$

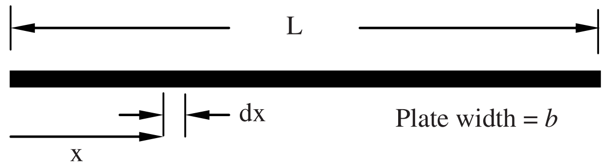
The above physical properties (k, c_p, ρ, μ) of the fluid can vary with temperature and are therefore evaluated at the mean film temperature

$$T_{film} = (T_s + T_\infty)/2$$

Where T_s is the plate is surface temperature and T_∞ is the temperature of the fluid in the farstream.

Equation 3.28 is applicable for Rex < 5x105 i.e. laminar flow and Pr ≥ 0.6 (air and water included).

In most engineering calculations, an overall value of the heat transfer coefficient is required rather than the local value. This can be obtained by integrating the heat transfer coefficient over the plate length as follows:



$$q_x = \text{(area of element) } h_x \ (T_s - T_\infty) = b\ dx\,(T_s - T_\infty) \qquad (3.29)$$

And for the plate entire length

$$Q = \int_0^L q_x = b\ L\ \overline{h_L}\ (T_s - T_\infty) \tag{3.30}$$

Where $\bar{h}_L$ is the average convective coefficient and Q is the total heat transfer from the plate. If we substitute for q_x from Equation 3.29 and rearrange we have,

$$\bar{h}_L = \frac{1}{L} \int_0^L h_x\ dx \tag{3.31}$$

If we now substitute for h_x from Equation 3.27 and integrate the resulting equation the average heat transfer coefficient is

$$\frac{\bar{h}_L L}{k} = 0.664\ \mathrm{Pr}^{1/3}\ \mathrm{Re}_L^{1/2} \tag{3.32}$$

or

$$\overline{Nu}_L = 0.664\quad \mathrm{Pr}^{1/3}\ \mathrm{Re}_L^{1/2} \tag{3.33}$$

This is valid for ReL < 5x105

Comparing Equation 3.28 with Equation 3.33 it can be seen that

$$\bar{h}_L = 2\ h_{\text{at x=L}}$$

Figure 3.7 shows a situation where there is an initial length of the flat plate which is not heated. The velocity boundary layer starts at $x = 0$ while the thermal boundary layer starts at $x = x_o$. In this case, it is possible to modify Equation 3.28 as follows:

$$Nu = \frac{0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1-\left(\frac{x_0}{x}\right)^{3/4}\right]^{1/3}} \quad (3.34)$$

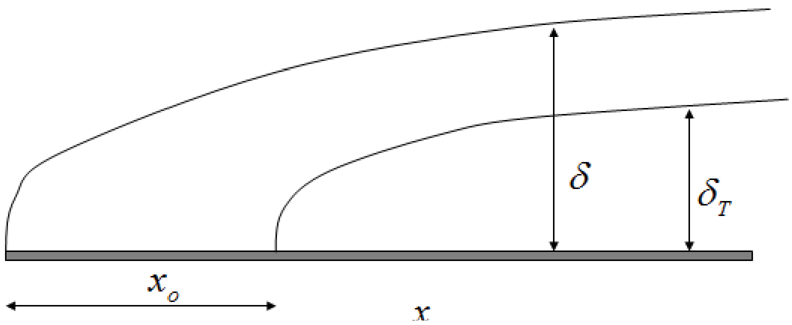


Figure 3-7: Thermal boundary layer starting at a different position from momentum boundary layer

3.4.2 Turbulent flow over a flat plate

Transition from laminar to turbulent flow over a flat plate generally takes place at a Reynolds number of approximately 5×10^5 . This value however can vary up to an order of magnitude either way dependent on the state of free stream turbulence and the smoothness of the plate. Figure 3.8 shows a schematic of the velocity profile near the wall for turbulent and laminar flow

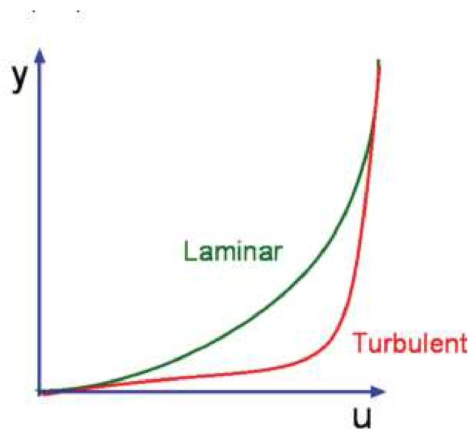


Figure 0-8 Velocity profiles for laminar and turbulent flows

It is apparent that the velocity gradient is much steeper for turbulent flow. By inspecting Equation 3.8, it is clear that for the same temperature difference between the wall and the free stream flow, turbulent flow will produce a larger heat transfer coefficient. Physically, this can be explained by the fact that there is more mixing of the flow due to turbulent fluctuations leading to a higher heat transfer.

For a flat plate, the Universal shear stress relationship for the velocity profile can be used, which is also known as the one-seventh power law

$$u^+ = 8.7(y^+)^{1/7} \qquad (3.35)$$

Which gives a good fit for experimental measurements.

Where $u^+ = \dfrac{u}{U^*}$, $y^+ = \dfrac{U^* y}{\nu}$ and $U^* = \sqrt{\dfrac{\tau_s}{\rho}}$ is know as the shear stress velocity.

Reynolds made an analogy between the momentum equation and energy equation, which lead to an expression of the Nusselt number as a function of the Reynolds number and wall shear stress as follows:

$$Nu_x = \tfrac{1}{2}\mathrm{Re}_x\, C_{fx} \qquad (3.36)$$

where

$$C_{fx} = \frac{\tau_s}{\frac{1}{2}\rho U_\infty^2} \qquad (3.37)$$

Equation 3.36 was obtained by imposing a condition of Pr = 1. However, since all fluids have a Pr which is different from unity, this equation was modified using empirical data using a factor of $\Pr^{1/3}$ which gives:

$$Nu_x = \tfrac{1}{2}\mathrm{Re}_x C_{fx} \Pr^{1/3} \qquad (3.38)$$

From Equation 3.35:

$$\frac{U_\infty}{U^*} = 8.7\left(\frac{U^*\delta}{\nu}\right)^{1/7} \qquad (3.39)$$

Dividing 3.35 by 3.39:

$$\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{1/7} \qquad (3.40)$$

Also, since $U^* = (\tau_s / \rho)^{1/2}$ then rearranging Equation 3.39 gives:

$$\frac{\tau_s}{\rho} = 0.0255 U_\infty^{7/4}\left(\frac{\nu}{\delta}\right)^{1/4} \qquad (3.41)$$

To obtain an expression for the shear stress, we need information about the growth of the boundary layer thickness δ . This can be found by integrating the momentum boundary layer Equation (see Long 1999) to obtain:

$$\frac{\delta}{x} = 0.37\,\mathrm{Re}_x^{-0.2} \qquad (3.42)$$

Substituting into Equation 3.36 gives:

$$C_{fx} = 0.0576\,\mathrm{Re}_x^{-0.2} \qquad (3.43)$$

Substituting into Equation 3.38 gives:

$$Nu_x = 0.0288\,\mathrm{Re}_x^{0.8} \Pr^{1/3} \qquad (3.44)$$

To compute the average Nusselt number for turbulent flow, it should be taken into account the a proportion of the blade will have a laminar boundary layer followed by transition before the flow becomes turbulent. So ideally, the Nusselt number needs to be integrated along the wall taking into account the change in the nature of the boundary layer. This leads to the following formula for the average Nusselt number (See Incropera and DeWitt 2002):

$$\overline{Nu_L} = \left[0.664\,\mathrm{Re}_{x,c}^{0.5} + 0.037\left(\mathrm{Re}_L^{0.8} - \mathrm{Re}_{x,c}^{0.8}\right)\right]\mathrm{Pr}^{1/3} \qquad (3.45)$$

Where $\mathrm{Re}_{x,c}$ is the Reynolds number at which transition occurs. Assuming that transition occurs at $\mathrm{Re}_L = 5\times10^5$, Equation 3.45 becomes:

$$\overline{Nu_L} = \left(0.037\,\mathrm{Re}_L^{0.8} - 871\right)\mathrm{Pr}^{1/3} \qquad (3.46)$$

If the length at which transition occurs is much smaller than the total length L of the plate, then Equation 3.46 can be approximated by

$$Nu_L = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3} \qquad (3.47)$$

Example 3.1

Air at temperature 527oC and 1 bar pressure flows with a velocity of 10m/s over a flat plate 0.5m long. Estimate the cooling rate per unit width of the plate needed to maintain it at a surface temperature of 27oC assuming the contribution of radiation contribution is negligible.

Solution:

The first step in to compute the heat transfer coefficient. This requires information allowing the computation of the Reynolds and Prandtl numbers.

For a large temperature difference between surface and fluid, the properties of air are evaluated at the mean film temperature (Section 3.4.1):

$$T_{film} = 0.5(27+527) = 277°C$$

From tabulated data, we obtain the following properties of air at 277oC and 1 bar:

$$\mu = 2.884\times10^{-5}\,Ns/m^2$$

$$\rho = 0.6329\,kg/m^3$$

$$c_p = 1040\,J/kgK$$

$$k = 0.044\,W/mK$$

The Reynolds number based on the plate length is then computed as:

$$\mathrm{Re}_L = \frac{\rho U_\infty L}{\mu} = \frac{0.6329 \times 10 \times 0.5}{2.884 \times 10^{-5}} = 109726$$

$\mathrm{Re}_L < 5 \times 10^5$ This means that the flow is laminar along the plate length, so we can use Equation 3.33 to calculate the average Nusselt number.

$$\mathrm{Pr} = \frac{\mu c_p}{k} = \frac{2.884 \times 10^{-5} \times 1040}{0.044} = 0.681$$

$$\overline{Nu}_L = 0.664 \quad \mathrm{Pr}^{1/3}\ \mathrm{Re}^{1/2} = 0.664 \times 0.681^{1/3} \times 109726^{1/2} = 193.5$$

$$\bar{h} = \frac{\overline{Nu}_L k}{L} = \frac{193.5 \times 0.044}{0.5} = 17 \quad W/m^2K$$

$$Q = hA(T_s - T_\infty) = 17 \times (0.5 \times 1)\left(527 - 27\right) = 4250 \quad W$$

This is the amount of heat that needs to be removed from the surface to keep it at a constant temperature of 27oC.

Example 3.2

A square flat plate of 2m each side is maintained at a uniform temperature of 230 oC by means of an embedded electric wire heater. If the atmospheric air is flowing at 25 oC over the plate with a velocity of 60m/s, what is the electrical power input required?

Solution:

Properties of air need to be evaluated at the mean film temperature of:

$$T_{film} = 0.5(275 + 25) = 127.5°C$$

From tabulated data, we obtain the following properties of air at 150oC (400K):

$$\mu = 2.301 \times 10^{-5}\, Ns/m^2$$
$$\rho = 0.871\, kg/m^3$$
$$c_p = 1014\, J/kgK$$
$$k = 0.0338\, W/mK$$

$$\mathrm{Re}_L = \frac{\rho U_\infty L}{\mu} = \frac{0.871 \times 60 \times 2}{2.301 \times 10^{-5}} = 45.422 \times 10^5$$

$\mathrm{Re}_L > 5 \times 10^5$ This means that the flow is turbulent over most of the plate length. Assuming that transition occurs at $\mathrm{Re}_L = 5 \times 10^5$, this means it will start at 0.22 m from the leading edge which is a small, but not negligible portion of the plate (just over 10%). We will use both Equations 3.46 and 3.47 to estimate the error resulting from the approximation in Equation 3.47

$$\mathrm{Pr} = \frac{\mu c_p}{k} = \frac{2.301 \times 10^{-5} \times 1013}{0.0338} = 0.690$$

From Equation 3.46

$$\overline{Nu_L} = \left(0.037\,\mathrm{Re}_L^{0.8} - 871\right)\mathrm{Pr}^{1/3} = \left(0.037 \times \left(45.422 \times 10^5\right)^{0.8} - 871\right) \times 0.690^{1/3} = 6153$$

$$\overline{Nu_L} = 0.036\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3} = 0.036 \times \left(45.422 \times 10^5\right)^{0.8} \times 0.690^{1/3} = 6923$$

Percentage error $\dfrac{6923 - 6153}{6153} \times 100 = 12.5\%$

Note that this error will reduce the increasing ReL.

Thus

$$\bar{h} = \frac{\overline{Nu}_L k}{L} = \frac{6153 \times 0.0338}{2.0} = 104 \quad W/m^2K$$

$$Q = hA(T_s - T_\infty) = 104 \times (2 \times 2)(230 - 25) = 85280 \quad W$$

Thus 75.28 kW of electric power needs to be supplied to keep the plate temperature.

3.4.3 Laminar flow in pipes

In Section 3.2.4, we discussed the nature of the flow in pipes and distinguished between two types of fully developed pipe flow, laminar and turbulent. If the Reynolds number Re_d , based on the pipe diameter is less than about 2300, then the flow is laminar. For Reynolds number above 2300 the boundary layer developing at the entrance of the pipe undergoes transition and becomes turbulent leading to a fully developed turbulent flow.

For laminar pipe flow, the velocity and temperature profiles can be derived from the solution of the flow Naveir-Stokes equations or boundary layer approximations leading to the determination of the heat transfer coefficient. We will not go it detailed derivations here and the interested reader can refer to Long (1999)

In pipe flow, we seek to determine a heat transfer coefficient such that Newton's law of cooling is formulated as:

$$q = h(T_s - T_m) \qquad (3.48)$$

Where T_s is the pipe-wall temperature and T_m is the mean temperature in the fully developed profile in the pipe. The mean fluid temperature is used instead of the free stream temperature for external flow.

In a similar fashion to the average Nusselt number for a flat plate, we define the average Nusselt number for pipe flow as: $\overline{Nu}_d = \bar{h}d/k$, were d is the pipe diameter.

From the derivation of formulae for the average Nusselt number for laminar follow resulting from the solution of the flow equations, it turns out the the Nusselt number is constant, and does not depend on either the Reyonlds or Prandtl numbers so long as the Reynolds number is below 2300.

However, two different solutions are found depending on the physical situation. For constant heat flux pipe flow, the Nusselt number is given by (see Incropera and DeWitt 2002):

$$\overline{Nu}_d = 4.36 \qquad (3.49)$$

While for a pipe with constant wall temperature, it is given by:

$$\overline{Nu}_d = 3.66 \qquad (3.50)$$

Note that to determine $\overline{h}$ from Equations 3.49 and 3.50, the heat transfer coefficient needs to be determined at the mean temperature T_m. For pipes were there is a significant variation of temperature between entry and exit of the pipe (such as in heat exchangers), then the fluid properties need to be determined at the arithmetic mean temperature between entry and exit (i..e at $(T_{m\ entry} - T_{m\ exit})/2$).

3.4.4 Turbulent flow in pipes

Determination of the heat transfer coefficient for turbulent pipe flow analytically is much more involved than that for laminar flow. Hence, greater emphasis is usually placed on empirical correlations.

The classic expression for local Nusselt number in turbulent pipe flow is due to Colburn, which is given by:

$$Nu_d = 0.023 \ \ \text{Re}_d^{4/5} \ \text{Pr}^{2/3} \qquad (3.51)$$

However, it is found that the Dittus-Boelter equation below provides a better correlation with measured data:

$$\overline{Nu}_d = 0.023 \ \ \text{Re}_d^{4/5} \ \text{Pr}^{n} \qquad (3.52)$$

where $n = 0.4$ for heating (Ts > Tm)

and $n = 0.3$ for cooling (Ts < Tm)

Equation 3.52 is valid for:

Red > 104, L / D > 10

$\left|T_s - T_f\right| < 5°C$ for liquids and

$\left|T_s - T_f\right| < 55°C$ for gasses

For larger temperature differences use of the following formula is recommended (Sieder and Tate, 1936).

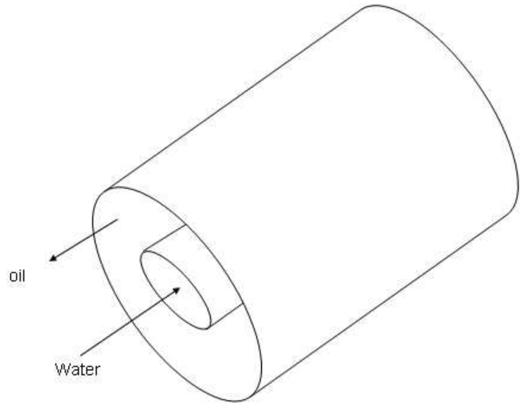
$$\overline{Nu}_d = 0.027 \ \ \text{Re}_d^{4/5} \ \text{Pr}^{1/3} \left(\frac{\mu}{\mu_s}\right)^{0.14} \qquad (3.53)$$

For $0.7 \le \text{Pr} \le 16700$, $\text{Re}_d \ge 10{,}000$ and $\frac{L}{d} \ge 10$

Where μ_s is the viscosity evaluated at the pipe surface temperature. The rest of the parameters are evaluated at the mean temperature.

Example 3.3

A concentric pipe heat exchanger is used to cool lubricating oil for a large diesel engine. The inner pipe of radius 30mm and has water flowing at a rate of 0.3 kg/s. The oil is flowing in the outer pipe, which has a radius of 50mm, at a rate of 0.15kg/s. Assuming fully developed flow in both inner and outer pipes, calculate the heat transfer coefficient for the water and oil sides respectively. Evaluate oil properties at 80oC and water properties at 35oC.



Solution

The first step is to obtain the relevant data from tables:

For oil at 80oC:

$c_p = 2131$ J/kg K, $\mu = 3.25 \times 10^{-2}$ Kg/m s, $k = 0.138$ W/m K

For water at 35oC:

$c_p = 4178$ J/kg K, $\mu = 725 \times 10^{-6}$ Kg/m s, $k = 0.625$ W/m K

The next step is to evaluate the Reynolds numbers:

$$\dot{m} = \rho V A$$

$$V = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{\rho \frac{\pi d^2}{4}}$$

$$\mathrm{Re}_d = \frac{\rho V d}{\mu} = \frac{4\rho\, \dot{m}\, d}{\rho\, \pi\, d^2\, \mu} = \frac{4\dot{m}}{\pi\, d\, \mu}$$

For water:

$$\mathrm{Re} = \frac{4 \times 0.3}{\pi \times 0.06 \times 725 \times 10^{-6}} = 8781$$

$$\mathrm{Pr} = \frac{\mu\, c_p}{k} \frac{725 \times 10^{-6} \times 4187}{0.625} = 4.85$$

Since Re > 2300, the flow is turbulent. Thus we can use Equation 3.52, with the exponent 0.4 as the water is being heated by the oil.

Thus

$$\overline{Nu_d} = 0.023\,\mathrm{Re}_d^{\,0.8}\,\mathrm{Pr}^{0.4} = 0.023 \times 8781^{0.8} \times 4.85^{0.4} = 62$$

and

$$\bar{h} = \frac{\overline{Nu_d}\,k}{d} = \frac{62 \times 0.625}{0.06} = 643 \text{ W/m2 K}$$

Oil is flowing in an annular shape pipe. We can use the same relations as a circular pipe, however, we use the hydraulic diameter instead of the diameter for calculating the Reynolds number. The hydraulic diameter is defined as:

$$d_h = \frac{4 \times \text{Area}}{\text{Wetter perimeter}}$$

Thus

$$d_h = \frac{4\pi(r_o^2 - r_i^2)}{2\pi(r_o + r_i)} = 2(r_o - r_i) = 2(0.05 - 0.03) = 0.036$$ m

$$V = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{\rho\,\pi(r_o^2 - r_i^2)}$$

$$\mathrm{Re}_d = \frac{\rho V d_h}{\mu} = \frac{2\rho\,\dot{m}(r_o - r_i)}{\rho\,\mu\,\pi(r_o^2 - r_i^2)} = \frac{2\dot{m}}{\pi\,\mu(r_o + r_i)}$$

$$\mathrm{Re}_d = \frac{2 \times 0.15}{\pi \times 3.25 \times 10^{-2}(0.05 + 0.03)} = 36$$

Since Re < 2300, the flow laminar. Thus we can use Equation 3.49, which gives a Nusselt number of 3.36 for pipe flow assuming a wall with a constant heat flux.

Then

$$\bar{h} = \frac{\overline{Nu_d}\,k}{d_h} = \frac{4.36 \times 0.138}{0.036} = 16.7$$ W/m2 K

Note that this is significantly lower than the water side heat transfer coefficient. If the pipe material is a good conductor, this then will be the limiting parameter in the heat exchange and to improve it, we need to increase the heat transfer coefficient. This for example can be done by for example increasing the flow velocity or reducing the outer diameter or both.

3.5 Natural convection

So far, we have discussed the process of heat convection in the presence of an external forcing condition, which we called forced convection. For example, fluid motion may be induced by a pump of a fan.

On the other hand, if fluid motion is caused by buoyancy forces within the fluid, it results in what we call natural or free convection. Buoyancy is due to the combined presence of a fluid density gradient and body force which is proportional to density.

An example of heat transfer by natural convection is that resulting from the external surface of radiators of a central heating system or an electric heating element. In this case, as the surrounding fluid is heated, its density reduces. This results in this fluid rising and will be replaced by colder fluid from the surrounding resulting in a circulation loop as shown in Figure 1.2.

Generally speaking, natural convection velocities are much smaller than those associated with forced convection resulting in smaller heat transfer coefficients.

If ρ_∞ is the density of the "undisturbed" cold fluid and ρ is the density of warmer fluid then the buoyancy force per unit volume F of fluid is:

$$F = (\rho_\infty - \rho) g \qquad (3.54)$$

Where g is the acceleration due to gravity.

The variation of density with temperature is:

$$\rho_\infty = \rho\,(1 + \beta\,\Delta T) \qquad (3.55)$$

where β is the volumetric thermal expansion coefficient ($1/K$) and ΔT is the temperature difference between the two fluid regions. If we substitute for ρ_∞ from Equation 3.54 into Equation 3.55 then the buoyancy force per unit volume of fluid is given as:

$$F = \rho\; g\; \beta\; \Delta T \qquad (3.56)$$

Therefore, in the case of natural convection, h could depend on a characteristic length L. A temperature difference ΔT, the conductivity k, the viscosity μ the specific heat capacity c_p, the density ρ, and the volumetric thermal expansion coefficient β of the fluid. β is usually grouped with g and ΔT as one term ($\beta g \Delta T$) as this group is proportional to the buoyancy force.

We will now use the principles of dimensional analysis discussed in section 1.3 to work out a set of non-dimensional parameters to group the parameters affecting natural convection. Thus

$$h = f(L, k, \mu, c_p, (\beta g \Delta T), \rho) \qquad (3.57)$$

These have the dimensions as shown in Table 3.2

Table 3-2: Parameters affecting natural convection and their dimensions

Parameter	Dimensions
h	$MT^{-3}\theta^{-1}$
$\beta g\Delta T$	$LT^{-2}\theta$
L	L
ΔT	θ
k	$MLT^{-3}\theta^{-1}$
μ	$ML^{-1}T^{-1}$
c_p	$L^2T^{-2}\theta^{-1}$
ρ	ML^{-3}

Again there are seven parameters and four dimensions which should lead to three non-dimensional groups.

Selecting the following repeated variables: $k,\quad L,\quad \mu,$ and ρ as they cannot form a non-dimensional group because only k has the dimension of temperature. We will then use the remaining variables as repeated variables one at a time. If we start with h, we get the following:

$$k^a L^b \mu^c \rho^d h = (MLT^{-3}\theta^{-1})^a (L)^b (ML^{-1}T^{-1})^c (ML^{-3})^d (MT^{-3}\theta^{-1}) = M^0 L^0 T^0 \theta^0$$

This is the same as Equation 3.23 which leads to the following non-dimensional group:

$$\pi_1 = \frac{hL}{k}$$

If we choose $\beta g \Delta T$, we get:

$$k^a L^b \mu^c \rho^d \beta g \Delta T = (MLT^{-3}\theta^{-1})^a (L)^b (ML^{-1}T^{-1})^c (ML^{-3})^d (LT^{-2}) = M^0 L^0 T^0 \theta^0$$

Then:

For M: $a + c + d = 0$
For T: $-3a - c - 2 = 0$
For θ: $-a = 0$
For L: $a + b - c - 3d + 1 = 0$

Solving these equations simultaneously gives:

$a = 0$, $b = 3$, $c = -2$, $d = 2$

This results in the following non-dimensional group:

$$\pi_2 = \frac{\rho^2 \beta\, g\, \Delta T\, L^3}{\mu^2} \qquad (3.58)$$

Repeating the procedure using the variable c_p leads to a third non-dimensional group as the case in forced convection:

$$\pi_3 = \frac{\mu\, c_p}{k}$$

Using the above non-dimensional groups, the functional relation in Equation 3.22 can be expressed as:

$$\frac{h\,L}{k} = f\left[\frac{\rho^2\ \beta\, g\ \Delta T\ L^3}{\mu^2}\ ,\frac{\mu\, c_p}{k}\right] \qquad (3.59)$$

or

$$Nu = f\left[Gr\ ,\Pr\right] \qquad (3.60)$$

where $$Gr = \frac{\rho^2 \beta g \Delta T L^3}{\mu^2}$$ is the Grashof number.

This dimensionless group is the ratio of the buoyancy forces to the square of the viscous forces in the fluid. Its role in natural convection is similar to the role of the Reynolds number in forced convection. At high Gr numbers the buoyancy forces are large compared to the viscous forces which tend to hold the fluid particles together and thus convection can occur.

In natural convection, we are also likely to encounter what is known as the Rayleigh number which is the product of he Grashof number and the Prandtl number:

$$Ra = Gr \Pr = \frac{g\beta \Delta T L^3}{\nu \alpha} \qquad (3.61)$$

This is the ratio of the thermal energy liberated by buoyancy to the energy dissipated by heat conduction and viscous drag.

It is customary to use the following expression for the Nusselt number in free convection, in a similar fashion to that used in natural convection:

$$Nu = C\, Gr^a \Pr^b \qquad (3.62)$$

where C, a and b are constants that can be determined either analytically experimentally, or using computational methods. Traditionally, experimental were methods used, but numerical procedures are becoming increasingly more used in recent years. Analytical methods are only possible for a limited number of very simple cases.

In the next few subsections we will present the formulae for various configurations encountered in engineering applications with some worked examples. In all cases, the fluid properties should be evaluated at $T_{film} = (T_s + T_\infty)/2$ for gasses $\beta = 1/T_{film}$.

3.5.1 Natural convection around horizontal cylinders

The terminology used is shown in Figure 3.9

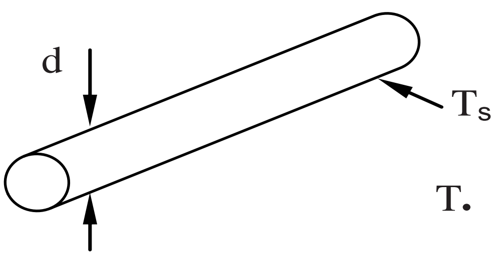


Figure 3-9: Natural convection around a horizontal cylinder

Churchill and Chu, 1975, provided the following correlation:

$$\overline{Nu}_d = \left\{ 0.6 + \frac{0.387\,(Ra_d)^{1/6}}{\left[1+(0.559/\Pr)^{9/16}\right]^{8/27}} \right\}^2 \qquad (3.63)$$

This equation is valid for Rad < 1012.

3.5.2 Natural convection around vertical cylinders

The terminology used is shown in Figure 3.10

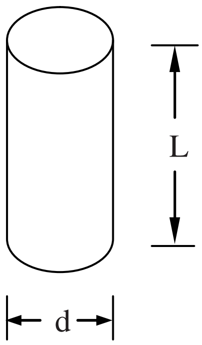


Figure 3-10 Natural convection around a vertical cylinder.

$$\overline{Nu}_d = 0.6 \left(Ra_d \frac{d}{L} \right)^{1/4} \qquad Ra_d \frac{d}{L} \geq 10^{1/4} \tag{3.64}$$

$$\overline{Nu}_d = 1.37 \left(Ra_d \frac{d}{L} \right)^{0.16} \qquad 0.05 \leq Ra_d \frac{d}{L} \leq 10^{1/4} \tag{3.65}$$

$$\overline{Nu}_d = 0.93 \left(Ra_d \frac{d}{L} \right)^{0.05} \qquad Ra_d \frac{d}{L} \leq 0.05 \tag{3.66}$$

3.5.3 Natural convection from flat plates

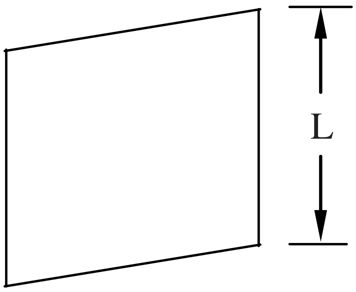


Figure 3-11 Natural convection from a vertical flat plate

For a vertical flat plate as shown in Figure 3.11, Churchill and Chu (1975) recommended the following equation for both laminar and also laminar and turbulent flow (entire range) respectively

For laminar flow:

$$\overline{Nu}_L = 0.68 + \frac{0.67\,(Ra_L)^{1/4}}{\left[1 + (0.492/\Pr)^{9/16}\right]^{4/9}} \qquad 0 < Ra_L < 10^9 \tag{3.67}$$

While the following formula gives the Nusselt number for the entire range of laminar and turbulent including the transition zone.

$$\overline{Nu}_L = \left\{ 0.825 + \frac{0.387\,(Ra_L)^{1/6}}{\left[1 + (0.492/\Pr)^{9/16}\right]^{8/27}} \right\}^2 \tag{3.68}$$

For a horizontal plate with a hot surface that faces upwards as shown in Figure 3.12, the equation suggested by Mc Adams (1954) and given here is widely used:

Figure 3-12 Natural convection over a horizontal flat plate

For laminar flow

$$\overline{Nu}_L = 0.54 \ (Ra_L)^{1/4} \qquad 10^4 < Ra_L < 10^7 \tag{3.69}$$

For turbulent flow

$$\overline{Nu}_L = 0.15 \ (Ra_L)^{1/3} \qquad 10^7 < Ra_L < 10^{11} \tag{3.70}$$

For a horizontal plate with a hot surface that faces downwards as shown in Figure 3.13:

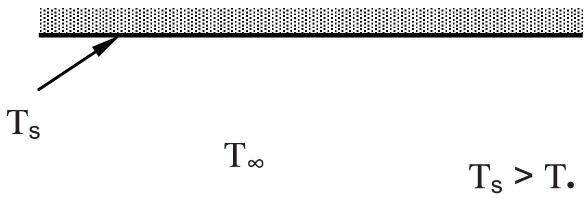


Figure 0-13Vertical plate facing downwards

$$\overline{Nu}_L = 0.27 \ (Ra_L)^{1/4} \qquad 10^5 < Ra_L < 10^{10} \tag{3.71}$$

Improvement in accuracy can be obtained if the characteristic length, L, in the Grashof and Nusselt number in Equations 3.69- 3.71 above is defined as:

$$L = \frac{\text{area of plate}}{\text{perimeter of plate}} \tag{3.72}$$

[Lloyd and Moran (1974) and Goldstein et al (1973)]

In the case of cold surface (Ts < T∞) Equations 3.69- 3.71 can be used for the surface facing downwards and upwards respectively.

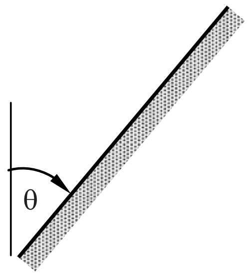


Figure 0-14 Natural convection from an inclined flat plate

For an inclined plate with an angle θ to the vertical as shown in Figure 3.14, use of Equations 3.69 and 3.71 is recommended as a first approximation for the top and bottom surfaces of cooled and heated plates respectively inclined up to 600 from the vertical with $g\cos\theta$ replacing g. For the opposite surfaces (bottom and top of cooled and heated plate), Incropera and Dewitt (2002) recommended that specific literature be consulted.

Example 3.4

A low pressure central heating radiator is simulated by a vertical flat plate 2 m wide and 1 m high. The water inlet and outlet temperatures are 85 and 75 0C respectively when the mass flow rate of water through the heater is 0.05 kg/s and the surrounding air is at 20 0C. Find the convective and the radiative component of heat transfer from the heater.

Solution

Data

c_p of water at (85 + 75)/2 = 4198 J/kg K

Assume that the surface temperature of the heater is

$$T_s = \frac{(85+75)}{2} = 80°C$$

$$T_{film} = \frac{(80+20)}{2} = 50°C = 323K$$

ρ = 1.093 kg/m3
μ = 1.953×10-5 kg/m s
k = 0.028 W/m K
Pr = 0.701

(From tables in Incropera and Dewitt, 2002)

Therefore

$$Gr_L = \frac{1.093^2 \times 9.81 \times (1/323) \times 60 \times 10^3}{(1.953 \times 10^{-5})^2} = 5.71 \times 10^9$$

and RaL is 5.71×109 × 0.701 = 4×109. Substitute in Equation 70

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387 \times (4 \times 10^9)^{1/6}}{[1+(0.492/0.701)^{9/16}]^{8/27}}\right\}^2 = 193.62$$

$\overline{Nu}_L = \overline{h}_L \ L / k_{film}$ therefore $\overline{h}_L$ = 193.62 × 0.028 / 1

$\overline{h}_L$ = 5.42 W/m2 K

And the convective component of the heat transfer from the heater is

$$Q_C = \bar{h}_L \times \text{total area} \times (T_s - T_{air})$$

$$Q_C = 5.42 \times 2 \times 2 \times (80 - 20) = 1301 \text{ W}$$

Total heat output

$$Q_t = 0.05 \times 4198 \times (85 - 75) = 2100 \text{ W}$$

Therefore the radiative component is

$$Q_r = Q_t - Q_c = 2100 - 1301 = 799 \text{ W}$$

3.6 Summary

In this chapter, the concept of the thermal boundary layer has been introduced after discussing the laminar and turbulent boundary layer and transition process.

To enable the analysis of the complex convection phenomena, dimensional analysis was introduced, where the number of controlling parameters is reduced to fewer sets of non-dimensional groupings. The main non-dimensional numbers relevant to convection are:

- Reynolds Number, which is the ratio of inertia and viscous forces
- Prandtl Number, which is the ratio of momentum diffusivity to thermal diffusivity
- Grashof Number, which is the ratio of buoyancy to the square of the viscous forces.
- Nusselt Number, which is the ratio of conductive to convective thermal resistance
- Rayleigh Number, which is the ratio of thermal energy liberated by buoyancy to the energy dissipated by heat conduction and viscous drag.

Empirical relations were developed which express the heat transfer coefficient as a function of one or more of those non-dimensional groups. Engineering applications were introduced where these relations were used to compute the convective heat transfer coefficient.

3.7 Multiple Choice Assessment

1. A heat transfer correlation is used to:
    - estimate Re
    - estimate the fluid velocity
    - estimate the fluid thermal properties
    - estimate the heat transfer coefficient
    - estimate radiation effects

2. Which of these is NOT a fluid property
    - density
    - thermal conductivity
    - viscosity
    - Prandtl number
    - Reynolds number

3. The Nusselt number is:
    - the ratio of inertial to viscous forces
    - the ratio of the thickness of the velocity and thermal boundary layers
    - a dimensionless heat transfer coefficient
    - another name for the European currency
    - the ratio of buoyancy to viscous forces

4. A value of Nu = 1 implies what ?
   - no heat transfer
   - the maximum possible heat transfer
   - heat transfer possible by radiation only
   - heat transfer possible by convection only
   - heat transfer possible by conduction only

5. Which statement is true of forced convection ?
   - $Nu \propto Gr\ Pr$
   - $Nu \propto Re\ Pr$
   - $Nu \propto Pr$ (only)
   - $Nu \propto M$ (Mach number)
   - $Nu \propto Re$ (only)

6. Which statement is true of free convection ?
   - h is always constant
   - h depends on external velocity
   - h depends on temperature difference, ΔT
   - h is independent of temperature difference, ΔT
   - the flow is always laminar

7. Which statement is true of forced convection ?
   - h is always constant
   - Nu = 1
   - h depends on temperature difference, ΔT
   - h is independent of temperature difference, ΔT
   - the flow is always turbulent

8. The definition of the Prandtl number is:
   - $Pr = \rho\ Cp / \mu$
   - $Pr = \mu\ Cp / \rho$
   - $Pr = \rho\ Cp / k$
   - $Pr = \mu\ Cp / k$
   - $Pr = h\ L / k$

9. In forced convection over a flat plate, what is the appropriate length scale for local values of Nu and Re ?
   - the boundary layer thickness, δ
   - the width (i.e. in the direction across the flow) of the plate
   - the thickness of the plate
   - the distance from the leading edge (i.e., in the direction of the flow), x
   - the overall length of the plate, L

10. In forced convection over a flat plate, what is the appropriate length scale for the average Nusselt number ?
    - the boundary layer thickness, δ
    - the width (i.e. in the direction across the flow) of the plate
    - the thickness of the plate
    - the distance from the leading edge (i.e., in the direction of the flow), x
    - the overall length of the plate, L

11. In forced convection over a cylinder, what is the appropriate length scale for local value of Nu and Re ?
    - the boundary layer thickness, δ
    - the length (i.e. in the direction across the flow) of the cylinder
    - the wall thickness of the cylinder
    - the diameter
    - your shoe size in cm.

12. The Rayleigh number, Ra is defined as:
    - Ra = Re Pr
    - Ra = Gr Re
    - Ra = Gr Pr
    - Ra = (Gr Pr)1/4
    - Ra = (Gr Pr)1/3

13. The Grashof number, Gr is defined as:

$$a)\ \frac{g\,\beta\,\Delta T\,L^3}{\mu^2};\ b)\ \frac{\rho^2\,g\,\beta\,\Delta T\,L^3}{\nu^2};\ c)\ \frac{g\,\beta\,\Delta T\,L^3}{\nu^2};\ d)\ \frac{g\,\beta\,\Delta T\,L}{\nu^2};\ e)\ \frac{U\,L}{\nu}$$

14. Which of the following is NOT a heat transfer correlation ?
    - Nux = 0.332 Rex1/2 Pr1/3
    - Nux = 0.0296 Rex0.8 Pr1/3
    - NuD = 0.023 ReD0.8 Pr1/3
    - NuD = 4.36
    - Nu = h L / k

15. In evaluating the Nusselt number, in for example, the flow of air over a steel body, the value of thermal conductivity used is:
    - 0.02 W / m K (i.e., air)
    - 30 W / m K (i.e., steel)
    - the average value of air and steel
    - it is not needed in the Nusselt number
    - it could be the fluid value or the surface value, but always the minimum of the two

16. If the local Nusselt number is given by Nux = 0.332 Rex1/2 Pr1/3, then the average Nusselt number is:
    - Nuav = 0.415 ReL0.8 Pr1/3
    - Nuav = 0.332 ReL1/2 Pr1/3
    - Nuav = 0.664 ReL1/2 Pr1/3
    - Nuav = 0.166 ReL1/2 Pr1/3
    - Nuav = 0.332 ReL0.8 Pr1/3

17. The correlation Nux = 0.54 Grx1/4 Pr1/4 apples to what ?
    - laminar forced convection from a plate
    - turbulent forced convection from a plate
    - laminar free convection from a vertical surface
    - turbulent free convection from a vertical surface
    - turbulent pipe flow

18. If the local Nusselt number is given by Nux = 0.54 Grx1/4 Pr1/4, then the average is:
    - Nuav = 0.27 GrL1/4 Pr1/4
    - Nuav = 0.405 GrL1/4 Pr1/4
    - Nuav = 0.54 GrL1/4 Pr1/4
    - Nuav = 0.72 GrL1/4 Pr1/4
    - Nuav = 1.08 GrL1/4 Pr1/4

19. The transition (from laminar to turbulent flow) Reynolds number for pipe flow is Re ≈ 2300, whereas for flow over a plate it is Re ≈ 2 x 105. What do you believe to be the reason for this difference ?
    - fluid friction is greater in a pipe
    - fluid friction is less in a pipe
    - the pressure gradient is negligible in the flow over a plate
    - the characteristic length scales are not the same
    - the structure of the turbulence is different in pipe flow to plate flow

20. Which of the following is the correct definition of the Reynolds number ?

$$\text{a) } \frac{\rho U L}{\mu};\ \text{b) } \frac{\rho U L}{\upsilon};\ \text{c) } \frac{U L}{\mu};\ \text{d) } \frac{\rho U L^2}{\mu};\ \text{e) } \frac{U}{(\gamma R T)^{1/2}}$$

21. If a local Nusselt number is given by: Nux = 0.331 Rex0.5, then this implies that the heat transfer coefficient is:
    - constant along the surface
    - increases linearly along the surface
    - increases with the square root of the distance along the surface
    - varies with the inverse square root of the distance along the surface
    - varies with the square of the distance along the surface

22. Which of the following non-dimensional groups characterises free convection ?
    - Reynolds number
    - Mach number
    - Weber number
    - Rayleigh number
    - Nusselt number

23. Which of the following correlations is the Dittus-Boelter equation ?
    - Nu = 0.387 (GrPr)1/4
    - Nu = 0.331 Re1/2 Pr1/3
    - NuD = 0.023 ReD0.8 Pr1/3
    - Nu = 0.662 Re1/2 Pr1/3
    - Nux = 0.023 Rex0.8 Pr1/3

24. The Prandtl number is a measure of
    - Compressibility effects
    - Turbulence level
    - Forced / Free convection effects
    - viscosity
    - relative thickness of velocity and thermal boundary layers

25. In a turbulent flow:
- the heat flux is less than in laminar flow
- the heat flux is greater than in laminar flow
- the heat flux is the same as in laminar flow
- the heat flux is zero
- the heat flux is infinite

26. An example of a body force is
- pressure
- friction
- inertia
- Coriolis force
- Buoyancy

27. At the outer edge ($y = \delta$) of a forced convection boundary layer:
- $u = 0$
- $v = 0$
- $\rho = 0$
- $q = 0$
- $\mu = 0$

28. At the outer edge ($y = \delta$) of a free convection boundary layer:
- $u = 0$
- $M = 1$
- $P = 0$
- $Gr = 0$
- $\rho = 0$

29. Given the heat transfer correlation Nux = 0.025 Rex0.8 Pr1/3, evaluate the local heat transfer coefficient for a fluid with k = 0.03 W/mK, μ = 3 x 10-5 kg/ms, Cp = 1kJ /kgK, ρ = 1.5 kg/m3 in a flow with a freestream velocity of 20 m/s and 2m downstream of the leading edge.
- 2746 W/m2K
- 27.46 W/m2K
- 373 W/m2K
- 41 W/m2K
- 3730 W/m2K

4. Radiation

4.1 Introduction

Heat transfer by thermal radiation, or infra red radiation, is a distinctly separate mechanism from convection and conduction. In particular, thermal radiation does not require any medium to transport thermal energy. So thermal radiation can travel through 'empty' space (like the heat reaching the earth's surface from the sun). Thermal radiation is an electromagnetic phenomenon, which occurs as a result of a bodies absolute temperature. So, all bodies with a temperature greater than absolute zero experience radiative heat transfer. Thermal radiation occurs in the range of wavelengths $0.1 < \lambda < 100$ μm (by way of comparison, visible light occupies the range $0.5 < \lambda < 1$ μm). For solid surfaces, thermal radiation is absorbed and emitted within about 1μm of the surface. It is therefore a surface effect and the properties which govern absorption and emission of thermal radiation depend on the surface characteristics (a layer of paint or other coating can for example significantly change the radiative properties). In general thermal radiation has both directional (depends on the angle relative to the surface) and spectral (depends on the wavelength) dependence. However, in this book we shall consider surfaces to be diffuse (they emit equally in all directions) and consider only total emission (which is the thermal radiation emitted across the entire spectrum). A further concept is that of a black body, which absorbs all incident radiation, reflects none and emits the maximum possible.

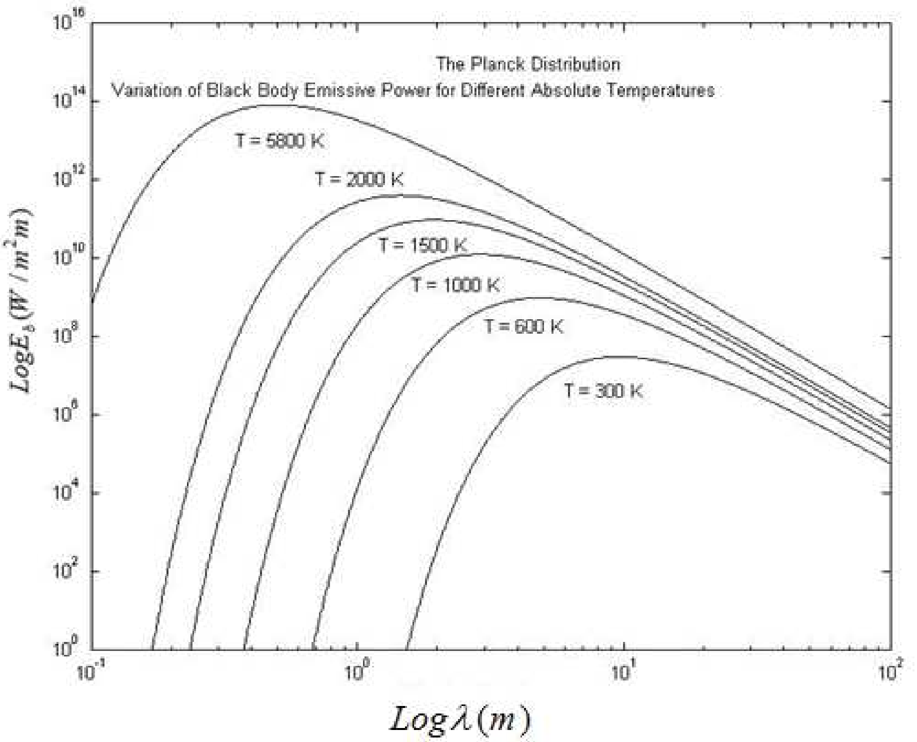


Figure 4.1: The Plank distribution for a black body

Figure 4.1 shows the spectral (variation across the spectrum or with wavelength) distribution of energy for a black body at various given temperatures, T. This is know as the Plank distribution. Note, for a given wavelength, λ, the energy emitted increases with increasing temperature; the peak in the energy distribution shifts towards shorter wavelengths as the temperature increases; for objects at a high temperature a significant fraction of the emission occurs in the visible part of the spectrum. The curves in Figure 4.1 are generated from the relationship below derived by Planck from considerations of quantum-statistical thermodynamics:

$$E_{\lambda,b} = \frac{2\pi h c^2}{\lambda^5 [\exp(hc/\lambda kT) - 1]} \quad (4.1)$$

where h is Planck's constant (6.6256 x 10-34 J s), c is the velocity of electromagnetic radiation in a vacuum (2.998 x 108 m/s) and k is the Boltzmann constant (1.3806 x 10-23 J/K).

Wien's displacement law is obtained by differentiating Equation (4.1) to find the wavelength at which the emission $E_{\lambda,b}$ is a maximum:

$$\lambda_{\max} T = \text{constant} = 2.8978 \text{ mm K} \quad (4.2)$$

Equation (4.2) can also be integrated over all wavelengths to give the heat transfer emitted by radiation from a black body, E_b as

$$E_b = \sigma T^4 \qquad (4.3)$$

where σ is the Stefan-Boltzmann constant (σ = 56.7 x 10-9 W/m2K4). A simple way of remembering the numerical value of this constant is the sequence of digits 5, 6, 7 and 8 since σ = 5.67 x 10-8 W/m2K4.

For two black bodies at temperatures T1 and T2, the maximum radiative interchange between them is given by:

$$E_{b,1} = \sigma T_1^4, \quad E_{b,2} = \sigma T_2^4, \quad E_{b,1-2} = \sigma (T_1^4 - T_2^4) \qquad (4.4)$$

4.2 Radiative Properties

Not all surfaces are ideal emitters and absorbers; they are not 'black' but 'grey'.

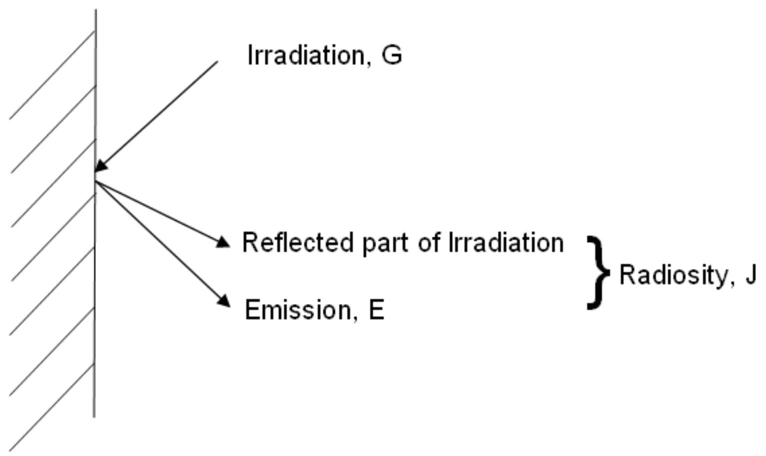


Figure 0-2: Thermal radiation from a surface

E: Emission is the thermal radiation emitted by a body and attributed to its absolute temperature.
G: Irradiation is the total thermal radiation incident on a surface.
J: Radiosity is the total radiation leaving a surface (Emission + reflected part of the Irradiation).

The units of emission, irradiation and radiosity are the same as a flux (W/m2). The magnitude of these fluxes depends not only on absolute temperature, but also a number of surface properties. These are:

the absorptivity, α
the transmissivity, τ
and
the reflectivity, ρ

Since the incident radiation may be absorbed, transmitted or reflected:

$$\alpha G + \tau G + \rho G = G \qquad (4.5)$$

or

$$\alpha + \tau + \rho = 1 \qquad (4.6)$$

Most solids are opaque $\tau = 0$; but not glass and some plastics. Since from Figure 4.2 $J = E + \rho G$, then for $\tau = 0$,

$$J = E + (1 - \alpha)G \qquad (4.7)$$

An additional property the emissivity, ε, is defined as: the radiation emitted by an actual surface / the radiation emitted by a black body at the same temperature. Hence:

$$\varepsilon = E / E_b \qquad (4.8)$$

Values of emissivity for some common surfaces are given in Table 4.1. Note the emissivity of a surface depends on the temperature at which emission occurs and also the surface finish.

Table 0-1 Typical values for total emissivity for some common different materials

Surface	*ε (range)*
Polished metals	*0.02 – 0.2*
Metals	*0.1 – 0.7*
Glass	*0.75 – 0.95*
Ceramic	*0.6 – 0.9*
Water	*0.9 – 0.95*
Wood	*0.8 – 0.9*
'Matt' paints	*0.9 – 0.98*

4.3 Kirchhoff's law of radiation

By considering a number of small bodies in a large enclosure, in a state of thermal equilibrium with each other and with the enclosure, it can be shown that:

$$\alpha = \varepsilon \qquad (4.9)$$

So, by knowing only ε (from, say tabulated data), we also know α ($\alpha = \varepsilon$) and for an opaque solid $\tau = 0$, so $\rho = 1 - \varepsilon$.

4.4 View factors and view factor algebra

Radiation analysis must take account of the fact that not all of one surface 'sees' all of another. This is characterised by the view factor (sometimes called the radiation configuration factor or shape factor)

The view factor, F, is defined as the fraction of radiation emitted from one surface that is incident upon another. It is usually given two subscripts, Fij, F12, Fab etc. The first subscript refers to the emitting surface the second the receiving surface. The mathematical definition of the view factors Fij and Fji are given by the expressions:

$$F_{ij} = \frac{1}{\pi A_i} \iint_{Aj\,Ai} \frac{\cos\theta_i \cos\theta_j dA_i \, dA_j}{r^2} \qquad (4.10a)$$

$$F_{ji} = \frac{1}{\pi A_j} \iint_{Ai\,Aj} \frac{\cos\theta_i \cos\theta_j dA_j \, dA_i}{r^2} \qquad (4.10b)$$

The above equations may be integrated to calculate view factors directly. In some cases, the integration can be simplified. View factors are also available for a large number of configurations in tabular, parametric or graphical form for a wide range of geometries. The Catalogue by Howell (1982) provides a comprehensive and useful source of view factor data.

From inspection of the symmetry between equations (4.10a) and (4.10b) it is apparent that

Ai Fij = Aj Fji (Reciprocity rule) (4.11)

Also for an enclosure of n surfaces:

$$\sum_{j=1}^{n} F_{ij} = 1 \qquad \text{(Summation rule)} \qquad (4.12)$$

For a convex or flat surface Fii = 0 (it does not 'see' any part of itself)
For a concave surface Fii > 0 (it does 'see' part of itself)

Example 4.1
The following examples illustrate a number of different techniques to calculate view factors.

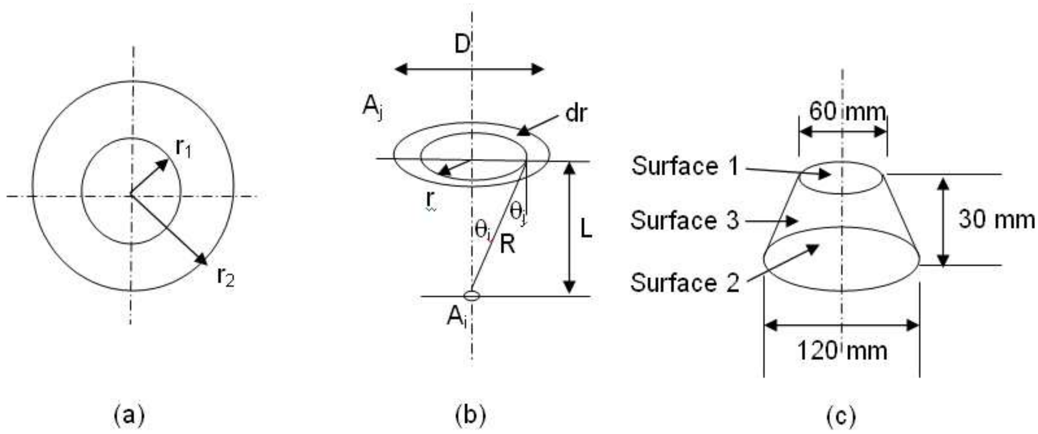


Figure 4-3: View factor geometries for Example 4.1

(a) This demonstrates the use of simple view factor algebra applied to two concentric cylinders.

F11 = 0 (convex surface)

so

F12 = 1

F21 = F12 (A1 / A2) = A1 / A2 = r1 / r2

and

F22 = 1 – (r1 / r2)

(b) This demonstrates how the complex view factor integral can sometimes be simplified. From Equation 4.10a

$$F_{ij} = \frac{1}{\pi A_i} \iint_{Aj\,Ai} \frac{\cos\theta_i \cos\theta_j dA_i \, dA_j}{R^2}$$

Since area Ai is small dAi = Ai and as a consequence θi, θj and r are independent of their position on the small area i. This implies that θi = θj = θ, from which:

$$F_{ij} = \frac{1}{\pi} \int_{A_j} \frac{\cos^2\theta \quad dA_j}{R^2}$$

Using the fact that R2 = L2 + r2, cosθ = L/R and dAj = 2πr dr

$$F_{ij} = 2\,L^2 \int_0^{D/2} \frac{r\;dr}{(L^2\,+\,r^2)^2} = \frac{D^2}{D^2\,+\,4L^2}$$

(c) This demonstrates the use of closed form parametric relationships to obtain a single view factor and the other values using view factor algebra. Note, the surface area, A, of a cone of base radius r, and perpendicular height, h is given by the expression: A = π r (r2 + h2)1/2. The surface area of a truncated cone (as shown here) is the difference between the surface area of the larger cone to that of the smaller one.

From Howell (1982), for two coaxial parallel discs of radius r1 and r2 separated by a distance a:

$$F_{12} = \frac{1}{2}\left\{X - \left[X^2 - 4(R_2 / R_1)^2\right]^{1/2}\right\}$$

where R1 = r1 / a; R2 = r2 / a; and X = 1 + { (1 + R22)/R12}

R1 = 1; R2 = 2; X = 6 and F12 = 0.763.

F11 = 0, so F13 = 1- 0.764 = 0.236

A1 = 900 π mm2; A2 = 3600 π mm2; A3 = 3818 π mm2
F21 = F12 (A1/A2) = 0.191

F22 = 0

F23 = 1 – 0.191 = 0.809

F31 = F13 (A1/A3) = 0.0556

F32 = F23 (A2/A3) = 0.763

F33 = 1 – 0.0556 – 0.763 = 0.181

4.4 Radiative Exchange Between a Number of Black Surfaces

There is no reflected component of radiation from a black surface (ρ = 0), the only energy to leave is by emission.

- Consider two radiatively black surfaces ‘i’ and ‘j’.
- The emission from surface i is σ Ti4, of which the fraction (Ai Fij) falls on surface j.
- Hence, the radiative heat transfer from i to j is: Qi-j = Ai Fij σ Ti4
- The emission from surface j is σ Tj4, of which the fraction (Aj Fji) falls on surface i.
- Hence, the radiative heat transfer from j to i is: Qj-i = Aj Fji σ Tj4

Since (reciprocity rule) Aj Fji = Ai Fij then the net radiation exchange between surface i and surface j is:

Qij = Qi-j - Qj-i = Ai Fij σ (Ti4 - Tj4) (4.12)

This can be easily generalised to the case of N black surfaces, which form an enclosure, as:

$$Q_i = \sum_{j=1}^{N} \sigma F_{ij} \; A_i \, (T_i^4 - T_j^4) \qquad (4.13)$$

4.5 Radiative Exchange Between a Number of Grey Surfaces

As shown in Figure 4.2, for a grey surface, the thermal radiation leaving the surface (the ‘radiosity’, J) comprises the reflected part of the radiation ρ G and the emission, εEb. Consider an enclosure formed from N separate grey surfaces at temperatures T1, T2, T3, …. TN. The respective areas are A1, A2, A3, …. AN, the respective emissivities are ε1, ε2, ε3, …. εN, and so on. Note: the particular geometry under consideration may not actually be fully enclosed (two oposing plates for example), any openings can be modelled as a hypothetical surface, approximating to the radiative properties of the surrounds. The net radiation leaving surface ‘i’, Qi , is equal to the difference between what goes out (the radiosity) and what goes in (the irradiation), so:

Qi = Ai (Ji – Gi) (4.14)

Also, and using Equation 4.8 for the definition of emissivity:

Ji = ρiGi + εiEb,i (4.15)

and for τ = 0 (opaque solid surface), ρ = (1-α) = (1 - ε) [Kirchhoff Equation 4.9]

Ji = (1 - εi)Gi + εiEb (4.16)

Rearranging Equation 4.14 for G and substituting in Equation 4.16 gives:

$$\frac{Q_i}{A_i} = \frac{E_{b,i} - J_i}{\dfrac{1-\varepsilon_i}{\varepsilon_i}} \qquad (4.17)$$

For our enclosure of N surfaces, the total radiation falling on surface ‘i’ due to the irradiation from all the other surfaces (including ‘i’) is Ai Gi. This quantity is equal to the sum of all the Aj Fji Jj. And using the reciprocity rule:

$$A_i G_i = \sum_{j=1}^{N} F_{ij} A_i J_j \tag{4.18}$$

Substituting (4.18) into (4.14), for G, gives:

$$Q_i = J_i - \sum_{j=1}^{N} F_{ij} J_i \text{ ,}$$

$$Q_i = A_i \sum_{j=1}^{N} F_{ij} (J_i - J_j) \tag{4.19}$$

Finally, combining Equations 4.19 and 4.17, which both express the heat flow in terms of radiosity gives a result that can be used directly:

$$Q_i = \frac{A_i (E_{b,i} - J_i)}{(1-\varepsilon_i)/\varepsilon_i} = A_i \sum_{j=1}^{N} F_{ij} (J_i - J_j) \tag{4.20}$$

How do we use this relationship ?

Apply Equation 4.20 to each surface in turn, e.g for a total of N = 3 surfaces: i = 1, j = 1, 2, 3; i = 2, j = 1, 2, 3; i = 3, j = 1, 2, 3. This will result in N (i.e. 3) simultaneous equations for the radiosities J1, J2 and J3, which can be eliminated and providing some temperatures are known, the other unknown temperatures may be found as may be the various heat flows. The following example will illustrate the practical application of this technique as well as how to deal with convective heat transfer in addition to radiation.

Example 4.2

High temperature gas flows through the inside of a pipe of outer radius r2 = 30 mm. To reduce the thermal radiation from the pipe to an electrical control panel mounted nearby, a semi-circular radiation shield of radius r1 = 100 mm is placed concentrically around the pipe. Thermal radiation from the pipe is radiated to bothe the shield and the surroundings which are at 310 K. The radiation view factor from the shield to itself is F11 = 0.3345.

a) Obtain values for the remaining view factors

b) Consider a radiation balance on the system. The pipe temperature is 900 K; the shield temperature is T1; the emissivity of the inner surface of the shield is ε1 = 0.8 and the emissivity of the outer surface of the pipe is ε2 = 0.5. Assume the surroundings can be approximated by a black body at 310 K and neglecting any heat loss by convection, estimate the surface temperature of the shield when the emissivity of the outer surface of the radiation shield is ε0 = 0.1

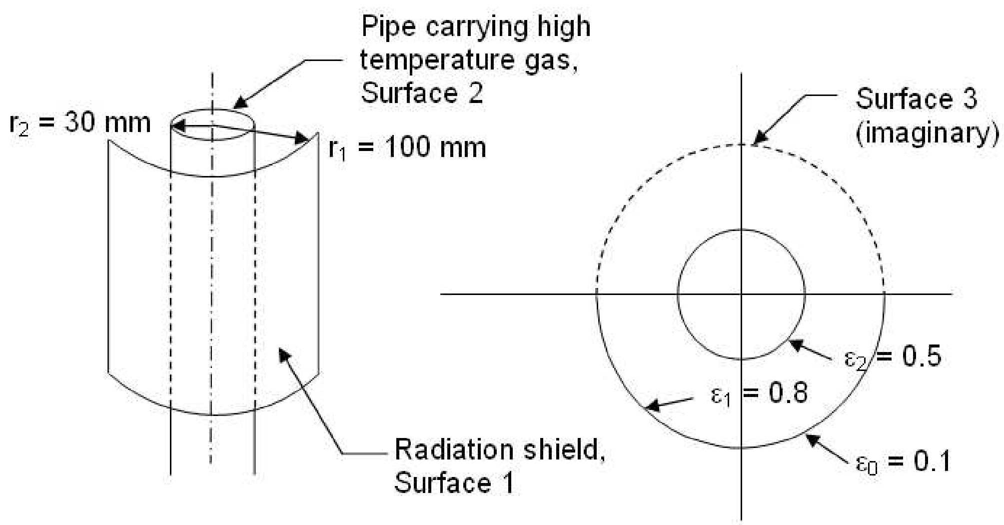


Figure 4.4 Radiation shield - Example 4.2

Figure 4.4 shows a cross-section through the pipe and radiation shield. A third, imaginary surface, Surface 3 is drawn to form the enclosure. The pipe and radiation shield are assumed to be long in relation to their diameters so it is a reasonable assumption to neglect radiation from the open ends.

Solution

a) Since F21 + F22 + F23 = 1, and F22 = 0, then by symmetry F21 = F23 = 0.5

From Example 4.1(a) F11 = 0.35

F12 = F21 A2 / A1= 0.15

F13 = 1 – F12 – F11 = 0.5

F32 = F23 A2 / A3 = 0.15

F33 = F11 (symmetry) = 0.35

F31 = 1 – F32 – F33 = 0.5

b) Using Equation (4.20), applied to Surface 1

$$\frac{E_{b,1} - J_1}{1 - \varepsilon_1 / \varepsilon_1} = F_{12}(J_1 - J_2) + F_{13}(J_1 - J_3)$$

Note the first term in the expansion of the series F11(J1 – J1) has been omitted because it is clearly zero.

Since ε1= 0.8, then 1 - ε1/ ε1 = 0.25, also Eb,1 = σT14 and if the surroundings, represented by Surface 3, can be approximated as a black body then J3 = σT33. After doing the algebra to isolate J1 and substituting the above quantities this gives

$$J_1 = \frac{\sigma T_1^4 + 0.25(F_{12}\, J_2 + \sigma\, F_{13}\, T_3^4)}{1 + 0.25(F_{12} + F_{13})}$$

Substituting F12 = 0.15 and F13 = 0.5 and with T3 = 310 K, σ = 56.7 x 10-9 W / m2 K4, this finally gives:

J1 = 4.877 x 10-8 T14 + 0.03226 J2 + 56.3 (W/m2)

Obviously this alone is insufficient to solve the problem, because we have three unknows J1, J2 and T1. A similar analysis must be carried out on Surface 2 (i.e. i = 2, j = 1,2,3).
This gives

$$\frac{E_{b,2} - J2}{1 - \varepsilon_2 / \varepsilon_2} = F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)$$

where 1 - ε2/ ε2 = 1, also Eb,2 = σT24 and J3 = σT33.

$$J_2 = \frac{\sigma T_2^4 + F_{21} J_1 + \sigma F_{23} T_3^4)}{1 + F_{21} + F_{23}}$$

Substituting F21 + F23 = 1, T2 = 900 K and σ = 56.7 x 10-9 W / m2 K4, gives

J2 = 18731 + 0.25 J1 (W/m2)

Combining the above two equations to eliminate J2 gives

J1 = 4.9 x 10-8 T14 + 666 (W/m2)

However, we have one equation with two unknowns so we need to use a further relationship. This comes from applying a heat balance on the inner and outer surfaces of the radiation shield itself. For the inside of the radiation shield

$$q_1 = \frac{E_{b,1} - J_1}{1 - \varepsilon_1 / \varepsilon_1}$$

and in the absence of losses by convection this heat flux leaves the outside surface of the radiation shield, so:

-q1 = ε0 σ(T14 – T34)

Combining these two expressions for q1 and substituting J1 = 4.9 x 10-8 T14 + 666, results in an equation with a single unknown quantity T1.

$$T_1^4 = \frac{0.025\sigma T_3^4 + 666}{\sigma(1+0.025) - 4.9\times10^{-8}}$$

From which, with T3 = 310 K, gives the final result that
T1 = 522 K.

It is instructive to continue with this example to examine the effect on the surface temperature of the radiation shield if we increase the emissivity of its outer surface and decrease the emissivity of its inner surface. Qualitatively we would expect the surface temperature to be reduced because less radiation from the hot gas pipe is being absorbed by the shield. If we now make ε0 = 0.8 and ε1 = 0.1, then we obtain:

J1 = 8.112 x 10-9 T14 + 0.193 J2 + 347.6 (W/m2)

J2 = 18731 + 0.25 J1 (W/m2) which is the same as before because ε1 is not involved in this equation.
When combined this gives

J1 = 8.534 x 10-9 T14 + 4163.5 (W/m2)

From then energy balance on the radiation shield

T1 = 465 K.

It is also useful to look at the effect of convective heat transfer, h, from the outer surface of the radiation shield. The energy balance on the outer surface of the shield becomes:

$$q_1 = \frac{E_{b,1} - J_1}{1 - \varepsilon_1 / \varepsilon_1} = -\varepsilon_0 \sigma (T_1^4 - T_3^4) + h(T_1 - T_3)$$

Taking h = 10 W/m2 K, T3 = 310 K, ε0 = 0.1 and ε1 = 0.8, gives

9.1175 x 10.9 T14 + 2.5T1 – 1453 = 0

which is satisfied for

T1 = 442 K

In determining what constitutes a surface it is necessary to recognise that these should be approximately isothermal. So, for example, in applying this technique to say a turbine disc where the temperature can vary significantly over the radius of the disc, it may be necessary to subdivide the disc into 10 or more sectors. This will of course result in a system of simultaneous equations that is no longer amenable to solving by hand and a computer matrix method will need to be used.

4.6 Radiation Exchange Between Two Grey Bodies

For the special case of two grey bodies, it is not necessary to use the radiosity-based simultaneous equation method described above. The following analytical expressions may be used depending on the particular configuration.

Two small bodies:

Q = ε1ε2 F12 A1 σ (T14 - T24) (4.21)

Two large parallel plates

$$Q = \frac{A \sigma (T_1^4 - T_2^4)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} \qquad (4.22)$$

Between a small grey body (surface 1) in a grey enclosure (surface 2)

Q = ε1 A1 σ (T14 - T24) (4.23)

4.7 Summary

Heat transfer by thermal, or infra-red, radiation occurs as a result of an objects' absolute temperature. It is an electromagnetic phenomenon which takes place in the range of wavelengths $0.1 < \lambda < 100$ μm. Therefore, and unlike conduction and convection, thermal radiation can propagate in the absence of any medium.

In addition to the thermal radiation emitted by an object, due to its temperature, thermal radiation may also be transmitted through a surface, reflected from it or absorbed into it. The physical properties known as emissivity, transmissivity, reflectivity and absorptivity determine the magnitude of these components. Values of these properties are easily obtained from tabulated data.

The idealised concept of a black body defines a perfect radiator and absorber. Real surfaces, which are termed "grey" emit and absorb less radiation than a black body. The radiation view factor determines the fraction of radiation leaving one surface that is incident upon another. For the general case, this results in a complex mathematical relationship. However, it is possible to simplify this for a limited number of geometries. For other more complex geometries the reader is referred to the catalogue produced by Howell (1982). View factor algebra is used to simplify the task of calculating all the view factors in a particular geometry.

A generalised method was developed for the analysis of grey body thermal radiative exchange in an enclosure. Most geometric configurations involving two or more surfaces can be made into an enclosure by the addition of an extra hypothetical surface that represents the surrounds.

Heat transfer by radiation always takes place when there is a temperature difference. Consequently, it is important to estimate the significance of the contribution of radiation compared with that for convection. In general, the heat flux by radiation will be significant in most free convection processes and in forced convection at high temperatures (for example, combustion).

4.8 Multiple Choice Assessment

1. Which of the following statements is true: Heat transfer by radiation ….
    - only occurs in outer space
    - is negligible in free convection
    - is a fluid phenomenon and travels at the speed of the fluid
    - is an acoustic phenomenon and travels at the speed of sound
    - is an electromagnetic phenomenon and travels at the speed of light

2. The value of the Stefan-Boltzmann constant is:
    - 56.7 x 10-9 W / K
    - 56.7 x 10-9 W / m2
    - 56.7 x 10-9 W / m K
    - 56.7 x 10-9 W / m2 K
    - 56.7 x 10-9 W / m2 K4

3. The range of wavelengths over which thermal radiation takes place is:
    - ≤ λ ≤ 100 nm
    - ≤ λ ≤ 100 μm
    - ≤ λ ≤ 100 mm
    - ≤ λ ≤ 100 m
    - ≤ λ ≤ 100 km

4. Which of the following statements applies to black body radiation
    - as the temperature increases the wavelength at which peak emission occurs decreases
    - as the temperature increases the wavelength at which peak emission occurs increases
    - as the temperature increases the wavelength at which peak emission occurs remains the same
    - as the temperature increases the peak emission shifts towards the infra-red
    - as the temperature increases the frequency at which the peak emission occurs decreases

5. The ratio: thermal radiation emitted by a surface to that emitted by a black body at the same temperature is known as:
   - reflectivity
   - radiosity
   - emissivity
   - solar irradiation
   - transmissivity

6. + α + ρ = ?
   - $\sqrt{-1}$; b) π ; c) 0 ; d) 1 ; e) –1

7. The view factor Fi,j is defined as:
   - the ratio of the emissivity of surface i to surface j
   - the ratio of the absolute temperature of surface i to surface j
   - the ratio of the area of surface i to surface j
   - the fraction of radiation emitted by surface j which is received by surface i.
   - the fraction of radiation emitted by surface i which is received by surface j.

8. The radiosity J is given by:
   - $J = \varepsilon E$
   - $J = \rho G$
   - $J = \rho G + \varepsilon Eb$
   - $J = \rho G + \varepsilon E$
   - $J = 1$

9. The reciprocity rule of view factor algebra states that:

a) $\sum_{j=1}^{N} A_i F_{i,j} = 0$; b) $\sum_{j=1}^{N} A_i F_{i,j} = 1$; c) $\sum_{j=1}^{N} F_{i,j} = 1$; d) $F_{i,j} = F_{j,i}$; e) $A_i F_{i,j} = A_j F_{j,i}$

10. 10.The summation rule of view factor algebra states that:

a) $\sum_{j=1}^{N} A_i F_{i,j} = 0$; b) $\sum_{j=1}^{N} A_i F_{i,j} = 1$; c) $\sum_{j=1}^{N} F_{i,j} = 1$; d) $F_{i,j} = F_{j,i}$; e) $A_i F_{i,j} = A_j F_{j,i}$

11. For a concave surface:
   - Fi,i = 0
   - Fi,i = < 0
   - Fi,i = > 0
   - Fi,i = 1
   - Fi,i = ∞

12. For a convex surface:
    - Fi,i = 0
    - Fi,i = < 0
    - Fi,i = > 0
    - Fi,i = 1
    - Fi,i = ∞

13. A1 = 1m2, F1,2 = 0.2, A2 = 2m2 and so F2,1 = ?
    - 1
    - 0.1
    - 0.2
    - 2
    - ∞

14. One side of a large marine boiler is maintained at 60°C in an environment at 30°C. The convective heat transfer coefficient is 10 W / m2 K. Assuming black body radiation the ratio of heat transfer by radiation to convection is:
    - 10
    - 0.1
    - 1.4
    - 0.7
    - 0.002

$a)\ \frac{E_{b,i} - J_i}{\frac{\varepsilon_i}{1-\varepsilon_i}} = \sum_{j=1}^{N} F_{i,j}\,(J_i - J_j)$

$b)\ \frac{E_{b,i} - J_i}{\frac{1-\varepsilon_i}{\varepsilon_i}} = \sum_{j=1}^{N} F_{i,j}\,(J_i - J_j)$

$c)\ \frac{E_{b,i} - J_i}{\frac{1-\varepsilon_i}{\varepsilon_i}} = \sum_{i=1}^{N} F_{i,j}\,(J_i - J_j)$

$d)\ \frac{E_i - J_i}{\frac{1-\varepsilon_i}{\varepsilon_i}} = \sum_{j=1}^{N} F_{i,j}\,(J_i - J_j)$

$e)\ \frac{E_{b,i} - J_i}{\frac{1-\varepsilon_i}{\varepsilon_i}} = \sum_{j=1}^{N} A_i F_{i,j}\,(J_i - J_j)$

15. 15.The correct formula for use with grey body radiative heat exchange is:

16. Kirchoff's law of radiation states that:
    - algebraic sum of the currents at a node is zero
    - Ji = Ei
    - εi = αi (whatever the temperature)
    - εi = αi (providing these processes take place at comparable temperatures)
    - Ei = Eb,i

17. Which of the following is the correct formulation of Wein's displacement law ?
    - λmaxT = 2.8978 mm K
    - λmaxT2 = 2.8978 mm K2
    - λmax / T = 2.8978 mm / K
    - λmax / T2 = 2.8978 mm / K2
    - λmax / T4 = 2.8978 mm / K4

18. The emissivity of a polished aluminium surface is (approximately) :
    - 0.002; b) 0.9; c) 0.7; d) 0.5; e) 0.1

19. The emissivity of a polished aluminium surface which is painted matt black is (approximately):
    - 0.002; b) 0.9; c) 0.7; d) 0.5; e) 0.1

21. In a four surface enclosure, how many view factors are there ?
    - 8
    - 4
    - 12
    - 15
    - 16

5. Heat Exchangers

5.1 Introduction

So far, we have studied the basic mechanisms for heat transfer. Various analytical and experimental correlations for the computation of the heat transfer were presented with numerous examples showing their application in engineering problems.

In this chapter, we will use those correlations to describe the calculation procedure for heat transfer within widely used heat transfer devices, namely heat exchangers. Heat exchangers are thermal devices that transfer or exchange heat from one fluid stream to one or more others. It is a broad description to a vast range of hardware that operates in one of three ways:

1. By recuperation, or recovery, of heat from a hot stream to a cold stream;

2. by regeneration, as the hot and cold streams alternatively flow through a matrix;

3. by direct contact of one fluid stream with another.

The emphasis in this book will be on recuperative heat exchangers. These being by far the most common but this analysis can be extended to the other types.

Examples of these types:

Recuperation: Automotive radiators, oil coolers, power station condensers, economiser.

Regenerative: Rotating matrix used to preheat exhaust gases, Stirling Cycle engine

Direct contact: Cooling of hot metal sheet by a spray of water or air jet, cooling tower.

In recuperative heat exchangers, mixing and contamination is prevented by solid walls. The dominant mechanisms of heat transfer are illustrated in Figure 5.1:

Convection: A → wall
Conduction: Through wall
Convection: wall → B

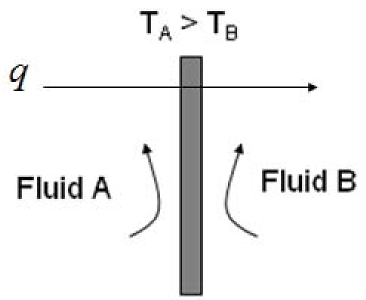


Figure 5-1 Mechanism of heat transfer in a heat exchanger

Radiation will be present as well, but in most cases (excluding combustion) it is safe to assume that this is also negligible.

There are at least two main objectives to the analysis of heat exchangers. The first is the design of a heat exchanger to fulfil a certain duty, providing geometric parameters. The second is the prediction of performance of a particular design working out inlet and outlet temperatures and heat flux.

5.2 Classification of Heat Exchangers

Heat Exchangers are classified either by flow arrangement, by construction or by their degree of compactness.

5.2.1 Classification by flow arrangement

In this classification, heat exchangers can be parallel flow, counter flow or cross flow as shown in Figure 5.2. In parallel flow heat exchangers, both flows run side by side, for example in concentric pipes. Counter flow heat exchangers are similar except the cold and hot streams go in opposite directions. Cross flow heat exchangers have the two flow stream normal to one another.

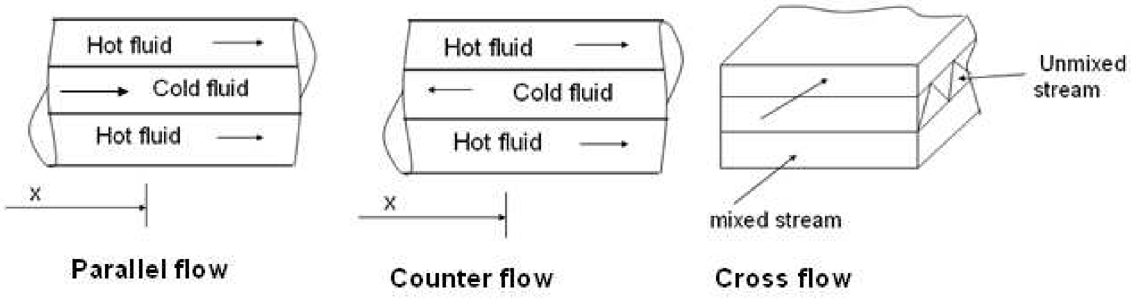


Figure 5-2 Classification of heat exchangers by flow arrangement

The temperature distributions in a parallel and cross flow heat exchanger as a function of the coordinate along the heat exchanger are given in Figure 5.3.

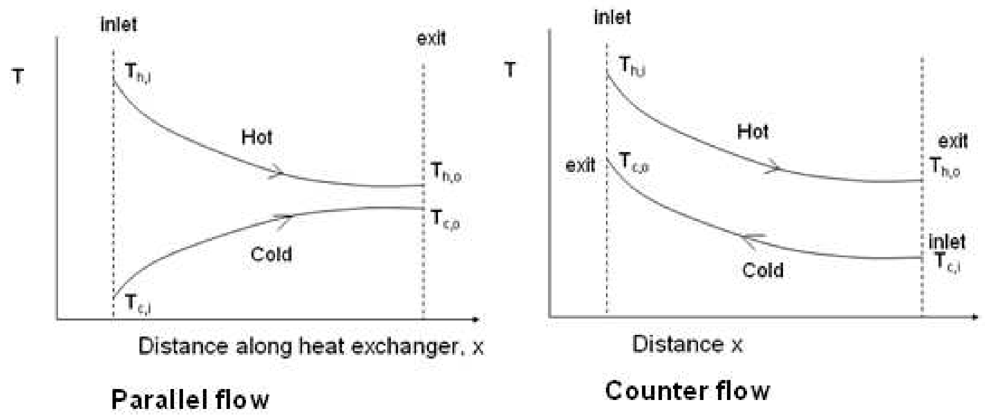


Figure 0-3 Temperature distribution in parallel and counter flow heat exchangers

It can be seen the exit temperature of the cold stream cannot by higher than the exit temperature of the hot stream for the parallel flow heat exchangers, while this is possible for counter flow heat exchangers.

5.2.2 Classification by construction

A double pipe heat exchanger has a relatively poor performance resulting in a very large physical size to perform a given duty consequently; they are rarely used in practical applications in this form. In practice, other arrangements are possible which allow reduction in the physical size of the heat exchanger. Examples are the U-tube bundle and the shell and tube heat exchangers shown in Figure 5.4

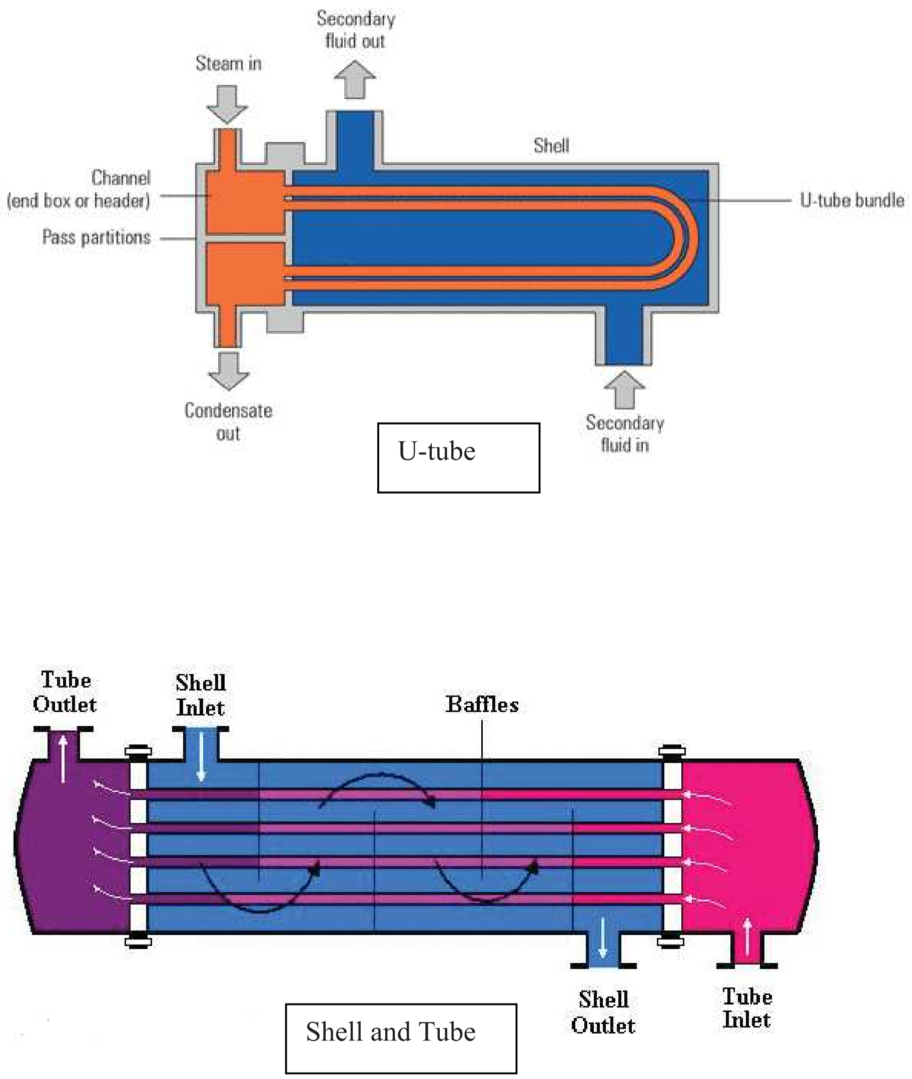


Figure 0-4 U-tube and shell-and-tube heat exchangers

A shell and tube is a development of a double pipe heat exchanger. This is a common form of construction, cheap and robust. However it is very heavy. Internal baffles are used to force the outer stream to cross the tubes, improving convective heat transfer.

A typical example of cross flow heat exchangers is the plate-Plate-and-fin heat exchanger. In these types, flow channels are constructed from parallel plates separated by fins. Fins are used in both sides in gas to gas applications and on the gas side for gas to liquid applications such as automotive radiators. These can be either mixed or unmixed flow heat exchangers depending on whether the flow across the tubes is separated into various channels by plates or not. Typical cross flow heat exchangers are shown in Figure 5.5.

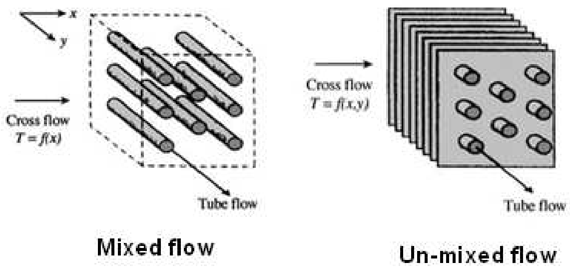


Figure 0-5 Examples of cross flow heat exchangers

5.2.3 Classification by compactness

Another way of classifying heat exchangers is the degree of compactness. This is expressed as the ratio the surface area to volume. Compact heat exchangers are devices offering high surface area to volume ratio. This can be achieved in a number of ways. They tend to be used in gaseous, rather than liquid applications where the heat transfer coefficients are low. Examples of compact heat exchangers are shown in Figure 5.6.

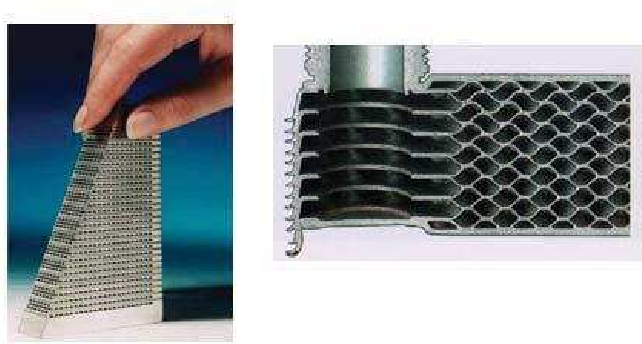


Figure 0-6 Examples of compact heat exchangers

5.3 The overall heat transfer coefficient

The overall heat transfer coefficient U is defined in terms of the total thermal resistance to heat transfer between two fluids. This implies that the Heat Transfer Q, area A and appropriate temperature difference are linked by the expression:

$$Q = UA\Delta T \qquad (5.1)$$

The coefficient is determined by accounting for conduction and convection resistance between fluids separated by solid walls. In heat exchanger analysis in this chapter, we will assume that the radiation effects are negligible compared to convection effects. This is true for most heat exchangers encountered in practical applications.

5.3.1 Overall heat transfer coefficient for a straight wall

The overall heat transfer coefficient is often the most uncertain part of any heat exchanger analysis. For a plane wall as shown in Figure 5.7, heat is transferred by convection on both sides and by conduction through the wall,

$$q = \frac{Q}{A} = h_h (T_h - T_1) \qquad (5.2)$$

$$q = \frac{k}{L}(T_1 - T_2) \qquad (5.3)$$

$$q = h_c (T_2 - T_c) \qquad (5.4)$$

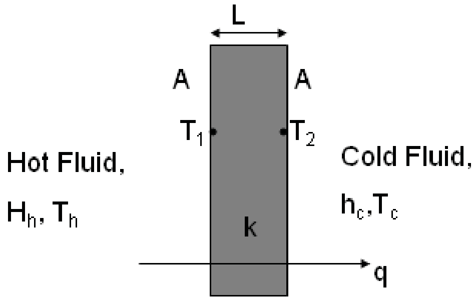


Figure 0-1: Heat flow across a solid wall with fluid either side

Rearranging Equations 5.2 to 5.4,

$$T_h - T_1 = \frac{q}{h_h} \qquad (5.5)$$

$$T_1 - T_2 = \frac{q}{k / L} \qquad (5.6)$$

$$T_2 - T_c = \frac{q}{h_c} \qquad (5.7)$$

Adding Equations 5.5 to 5.7 to eliminate wall temperatures gives:

$$T_h - T_c = q\left[\frac{1}{h_h} + \frac{L}{k} + \frac{1}{h_c}\right] \qquad (5.8)$$

which can be rearranged to give:

$$q = \frac{T_h - T_c}{\left[\frac{1}{h_h} + \frac{L}{k} + \frac{1}{h_c}\right]} = U(T_h - T_c) \qquad (5.9)$$

Thus U is the overall heat transfer coefficient, referred to also as the U-value.

$$U = \frac{1}{\left[\frac{1}{h_h} + \frac{L}{k} + \frac{1}{h_c}\right]} \tag{5.10}$$

In the general case, the area on the hot and cold sides might not be equal, so the area needs to be taken into account. We then can define the overall heat transfer coefficient in that context as:

$$UA = U_h A_h = U_c A_c = \frac{1}{\left[\frac{1}{(hA)_h} + R_w + \frac{1}{(hA)_c}\right]} \tag{5.11}$$

So if the area at the hot side is not the same as that on the cold side, then the associated heat transfer coefficient will not be the same.

5.3.2 Overall heat transfer coefficient for circular pipe

For heat transfer from concentric circular pipes, which is a common encounter in heat exchangers, we will use the schematic of Figure 5.7 to perform an analysis leading to working out the overall heat transfer coefficient.

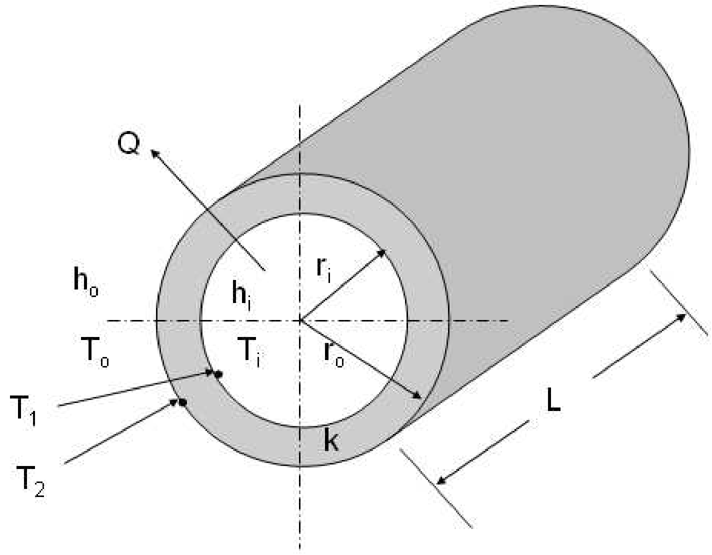


Figure 0-2: Heat transfer through concentric circular pipes

Heat flow Q through the pipe is given by:

$$Q = A_i h_i (T_i - T_1) = 2\pi\, r_i\, L\, h_i\, (T_i - T_1) \qquad (5.12)$$

and from section 2.2:

$$Q = -2\pi\, r\, L\, K \frac{dT}{dr} \qquad (5.13)$$

$$Q = A_o h_o (T_2 - T_o) = 2\pi\, r_o\, L\, h_o\, (T_2 - T_o) \qquad (5.14)$$

Rearranging Equation 5.12 and integrating:

$$dT = -\left(\frac{Q}{2\pi kL}\right)\frac{dr}{r} \qquad (5.15a)$$

$$T_1 - T_2 = \frac{Q}{2\pi kL}\ln\left(\frac{r_o}{r_i}\right) \quad (5.15b)$$

Rearranging Equations and 5.12 and 5.14 gives:

$$T_i - T_1 = \frac{Q}{2\pi\, r_i L h_i} \quad (5.16)$$

$$T_2 - T_o = \frac{Q}{2\pi r_o L h_o} \quad (5.17)$$

Adding Equations 5.15b to 5.17 to eliminate the wall temperatures gives:

$$T_i - T_o = \frac{Q}{2\pi L}\left[\frac{\ln(r_o / r_i)}{k} + \frac{1}{h_i r_i} + \frac{1}{h_o r_o}\right] \quad (5.18)$$

Rearranging:

$$\frac{Q}{2\pi L r_o} = \frac{T_i - T_o}{\left[\frac{r_o}{k}\ln\left(\frac{r_o}{r_i}\right) + \frac{r_o}{h_i r_i} + \frac{1}{h_o}\right]} = U_o (T_i - T_o) \quad (5.19)$$

Where U_o is the overall heat transfer coefficient based on the outer area of the inner tube:

$$U_o = \left[\frac{r_o}{k}\ln\left(\frac{r_o}{r_i}\right) + \frac{r_o}{h_i r_i} + \frac{1}{h_o}\right]^{-1} \quad (5.20)$$

Similarly we can work out an overall heat transfer coefficient based on the inner area of the inner tube:

$$U_i = \left[\frac{r_i}{k}\ln\left(\frac{r_o}{r_i}\right) + \frac{r_i}{h_o r_o} + \frac{1}{h_i}\right]^{-1} \quad (5.21)$$

Typical Values of overall heat transfer coefficients are given in Table 5.1

Table 0-1 Typical values of overall heat transfer coefficients

Hot fluid	Cold Fluid	U (W/m2K)
Water	Water	1000-2500
Ammonia	Water	1000-2500
Gases	Water	10-250
Steam	Water	1000-3500
Steam	Gases	25-250

If $r_o/r_i \approx 1$ and the tubes are made from a good conductor, then the overall heat transfer coefficient reduces to:

$$U_i = U_o = \left[\frac{1}{h_i} + \frac{1}{h_o}\right]^{-1} = \frac{h_o h_i}{h_o + h_i} = U \qquad (5.22)$$

if $h_i >> h_o$, then $U \approx h_o$

if $h_o >> h_i$, then $U \approx h_i$

So the heat transfer coefficient is controlled by the lower of heat transfer coefficients. To illustrate this assume a water to air heat exchanger, where the air side heat transfer coefficient is 40 W/m2K and the water side is 1000 W/m2K:

$$U = \frac{40 \times 1000}{40 + 1000} = 38.46 W / m^2 K$$

Consider two cases, one where the water side heat transfer coefficient is doubled and the other where the air side is doubled:

$$U = \frac{40 \times 2000}{40 + 2000} = 39.21 W / m^2 K$$

$$U = \frac{80 \times 1000}{80 + 1000} = 74.07 W / m^2 K$$

The former leads to a 2% change while the latter leads to 93% change.

Example 5.1

Using the Dittus-Boelter correlation (Equation 3.52):

$$Nu_D = 0.023 \, \mathrm{Re}_D^{0.8} \, \mathrm{Pr}^n$$
$$n = 0.3 \qquad \text{cooling} \qquad (T_s < T_f)$$
$$n = 0.4 \qquad \text{heating} \qquad (T_s > T_f)$$

Calculate the heat transfer coefficient for:

1 kg/s flow of water, density, 1000 kg/m3 at 300 K, k = 0.6 W/m K, Viscosity of 0.001 kg/ms, in a 50mm diameter tube, Pr=6 (Hot tube, cold fluid).

For air, but with a velocity of fifteen times that if the water in part a. For air take the density 1.2 kg/m3, k = 0.02 W/m K and viscosity of 1.8x10-5 kg/ms, Pr = 0.7

Solution:

(a)

$$\dot{m} = \rho V A$$

$$V = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{\rho \frac{\pi d^2}{4}} = \frac{4 \times 1}{10^3 \frac{\pi \times 0.05^2}{4}} = 0.51 m / s$$

$$\mathrm{Re}_d = \frac{\rho V d}{\mu} = \frac{4 \dot{m}}{\pi \, d \, \mu} = \frac{4 \times 1}{\pi \times 0.05 \times 10^{-3}} = 25464$$

$$Nu_d = 0.023 \times 25464^{0.8} \times 0.6^{0.4} = 158$$

$$Nu_d = \frac{h\,d}{k}$$

$$h = \frac{Nu_d k}{d} = \frac{158 \times 0.6}{0.05} = 1892 W/m^2 K$$

(b)

$$V = 15 \times 0.51 = 7.65 m/s$$

$$\mathrm{Re}_d = \frac{1.2 \times 7.65 \times 0.05}{1.8 \times 10^{-5}} = 25500$$ (Fifteen times velocity in a , but similar Re)

$$Nu_d = 0.023 \times 25500^{0.8} \times 0.7^{0.4} = 67$$

$$h = \frac{67 \times 0.02}{0.05} = 27 W/m^2 K$$

(Similar values of Re but radically different h).

5.4 Analysis of Heat Exchangers

There are two methods in use for the analysis of heat exchangers. The first is called the Log Mean Temperature Difference method (LMTD) and the second is called the Effectiveness – NTU method (Number of transfer Units) or simply the NTU method. In this book, we will explain and use the LMTD method. For the NTU method, the reader is referred to either Long (1999) or Incropera and DeWitt (2002).

The heat transfer from one fluid stream to another can be written as:

$$Q = UA\Delta T_m \qquad (5.23)$$

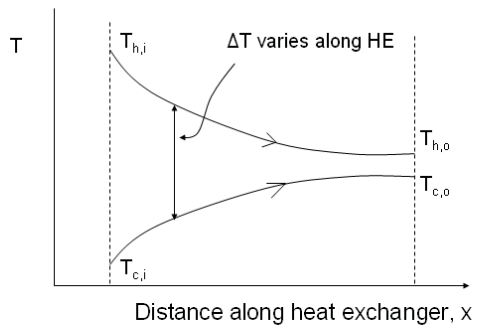


Figure 0-3: Temperature distribution for a parallel flow heat exchanger

ΔT_m : Some mean temperature difference, which will be defined later.

To determine ΔT_m we will make the following assumptions:

- No external losses from the heat exchanger
- Negligible conduction along the tube length;
- Changes in kinetic and potential energy are negligible
- h is constant along the length of the heat exchanger
- Specific heats are constant (not a function of temperature).

Consider a parallel flow heat exchanger with the temperature distribution as shown in Figure 5.9:

The general relationship for the heat transfer from one fluid stream to the other is given by:

$$Q = \dot{m} c_p (T_{Fluid-in} - T_{Fluid-out}) \qquad (5.24)$$

$$Q_H = \dot{m}_H c_{pH} (T_{Hi} - T_{Ho})$$

$$Q_C = \dot{m}_C c_{pC}(T_{Co} - T_{Ci})$$

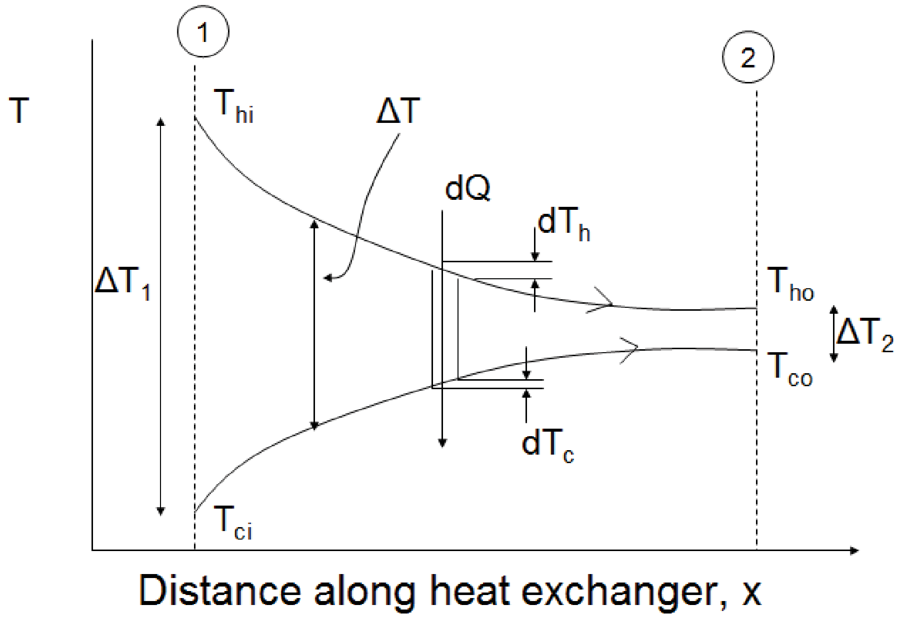


Figure 0-4: Analysis of heat flow through parallel flow heat exchanger

For the elemental section shown in Figure 5.9:

$$dQ_H = -dQ_C = dQ \qquad (5.25)$$

$$dQ_H = \dot{m}_H C_{pH} dT_H \qquad (5.26)$$

$$dQ_C = \dot{m}_C C_{pC} dT_C \qquad (5.27)$$

The overall change in temperature difference across the element is given by:

$$d(\Delta T) = dT_H - dT_C \qquad (5.28)$$

$$d(\Delta T) = -dQ\left[\frac{1}{\dot{m}_H C_{pH}} + \frac{1}{\dot{m}_C C_{pC}}\right] \qquad (5.29)$$

From Equation 5.1 we know that

$$Q = UA\Delta T \qquad \rightarrow \qquad dQ = UdA\Delta T \qquad (5.30)$$

which gives

$$\frac{dQ}{dA} = U\Delta T \qquad (5.31)$$

Where $\Delta T = T_H - T_C$ at any point

Combining Equations 5.29 and 5.30 to eliminate dQ :

$$\left[\frac{d\Delta T}{\dfrac{1}{\dot{m}_H C_{pH}} + \dfrac{1}{\dot{m}_C C_{pC}}}\right] = -U\Delta T dA \qquad (5.32)$$

Rearranging:

$$\frac{d\Delta T}{\Delta T} = -U\left[\frac{1}{\dot{m}_H C_{pH}} + \frac{1}{\dot{m}_C C_{pC}}\right] dA \qquad (5.33)$$

Integrating from point 1 at inlet to point 2 at exit:

$$\int_1^2 \frac{d\Delta T}{\Delta T} = -U\left[\frac{1}{\dot{m}_H C_{pH}} + \frac{1}{\dot{m}_C C_{pC}}\right] \int_1^2 dA \qquad (5.34)$$

For the parallel flow heat exchanger:

$$\Delta T_1 = T_{h,i} - T_{c,i} \qquad (5.35)$$

$$\Delta T_2 = T_{h,o} - T_{c,o} \qquad (5.36)$$

Integrating 5.34 and substituting $\dot{m}C_p = Q / \Delta T$:

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -\frac{UA}{Q}\left(T_{h,i} - T_{h,o} + T_{c,o} - T_{c,i}\right) = -\frac{UA}{Q}(\Delta T_1 - \Delta T_2) \qquad (5.37)$$

Rearranging:

$$Q = UA\left[\frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}\right] = UA\Delta T_m \tag{5.38}$$

Where ΔT_m is the Log Mean Temperature Difference (LMTD) defined as:

$$\Delta T_m = \left[\frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}\right] \tag{5.39}$$

For a counter flow heat exchanger, with a temperature distribution as shown in Figure 5.10:

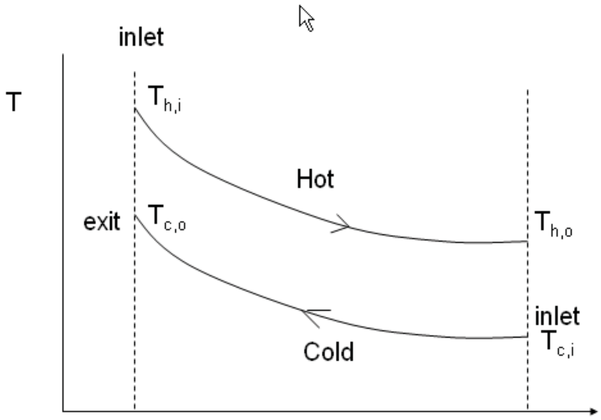


Figure 0-5: Counter flow heat exchanger terminology

If we define:

$$\Delta T_1 = T_{h,i} - T_{c,o} \qquad (5.40)$$

$$\Delta T_2 = T_{h,o} - T_{c,i} \qquad (5.41)$$

And we repeat the derivation; an identical expression is obtained for the LMTD as in Equations 5.38 and 5.39. However, care should be taken when defining the temperature difference in Equations 5.40 and 5.41 as opposed to those for parallel flow heat exchanger given in Equations 5.35 and 5.36.

Equations 5.38 and 5.39 can also be extended to other types of heat exchangers such as cross flow or shell and tube using a correction factor which is a function of two other dimensionless factors which are in turn defined empirically as follows:

$$\Delta T_m = \left(\Delta T_{1m}\right)_{cf} \times F \qquad (5.42)$$

Where $\left(\Delta T_{1m}\right)_{cf}$ is the log mean temperature difference assuming counter flow heat exchanger and F is the correction factor defined as follows:

$$F = f(P,R) \qquad (5.43)$$

Where P and R are empirical parameters defined as follows:

$$R = \frac{T_1 - T_2}{t_2 - t_1}, \qquad P = \frac{t_2 - t_1}{T_1 - t_1} \tag{5.44}$$

The temperatures in Equation 5.44 are defined, for example for a cross flow heat exchanger as shown in Figure 5.11.

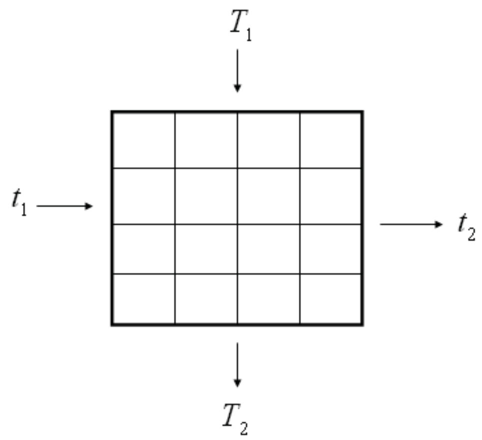


Figure 0-6 Inlet and outlet temperatures for a cross flow heat exchanger

R and P can be obtained from the empirical charts for various configurations. Figure 5.12 shows the correction factor for single pass cross flow heat exchanger with both streams unmixed. Correction factors for other configurations can be found in Incroperal and DeWitt (2002).

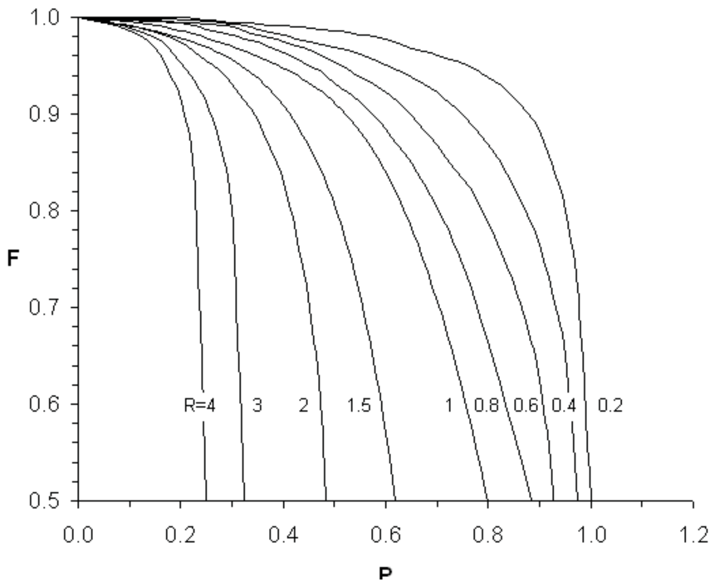


Figure 0-7: Correction factor F for a single-pass cross flow heat exchanger with both streams unmixed

Example 5.2

A concentric tube heat exchanger is used to cool lubricating oil for a large diesel engine. The inner tube is constructed of 2 mm wall thickness stainless steel, having k = 16 W/m K. The flow rate of cooling water through the inner tube (ri = 30mm) is 0.3 kg/s. The flow rate of oil through the tube (ro = 50mm) is 0.15 kg/s. Assume fully developed flow, if the oil cooler is to be used to cool oil from 90oC to 50oC using water available at 10oC, calculate:

The length of the tube required for parallel flow;

The length of the tube required for counterflow;

The area required for a single pass cross-flow heat exchanger with both streams umixed, operating at the same temperatures and flow rates and with the same value of U as in a and b above.

Solution:

The water temperature at exit is unknown. This can be computed from the overall energy balance for oil and water.

For oil:

$$Q = \dot{m}_h C_{p,h} (T_{h,i} - T_{h,o}) = 0.15 \times 2131(90-50) = 12786\, W$$

$$Q = \dot{m}_c C_{p,c} (T_{c,i} - T_{c,o})$$

$$12786 = 0.3 \times 4178(10 - T_{c,o})$$

$$T_{c,o} = 20.2^o C$$

For parallel flow:

$$\Delta T_1 = (90-10) = 80^o C$$

$$\Delta T_2 = (50-20.2) = 29.8^o C$$

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)} = \frac{29.8-80}{\ln(29.8/80)} = 50.83^o C$$

For counter flow:

$$\Delta T_1 = (90-20.2) = 69.8^o C$$

$$\Delta T_2 = (50-10) = 40^o C$$

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)} = \frac{40-69.8}{\ln(40/69.8)} = 53.52^o C$$

For cross, flow, both streams are unmixed, in terms of the nomenclature for the cross flow:

$$t_1 = T_{h,i} = 90^o C,\ t_2 = T_{h,o} = 50^o C,\quad T_1 = T_{c,i} = 10^o C,\ T_2 = T_{c,o} = 20.2^o C$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} = 0.255 \qquad P = \frac{t_2 - t_1}{T_1 - t_1} = 0.5$$

From Figure 5.12, $F \approx 0.98$

Therefore:

$$\Delta T_m = \Delta T_{lm,cf} F = 53.52 \times 0.98 = 52.45^o C$$

Area of tube:

$$Q = UA\Delta T_{lm}$$

$$A = \frac{Q}{U\Delta T_{lm}}$$

For parallel flow:

$$A = \frac{12786}{21.9 \times 50.83} = 11.2m^2$$

$$L = \frac{A}{2\pi r_i} = \frac{11.2}{2\pi \times 0.03} = 61m$$

For counter flow:

$$A = \frac{12786}{21.9 \times 53.52} = 10.9m^2$$

$$L = \frac{A}{2\pi r_i} = \frac{10.9}{2\pi \times 0.03} = 57.9m$$

For cross flow:

$$A = \frac{12786}{21.9 \times 52.45} = 11.13m^2$$

Comment:
The counter flow heat exchanger has the smallest area and thus can be considered more efficient. However, both parallel and counter flow are very long and could be impractical for engineering applications. The cross flow might provide the area required in a shorter length, however, a shell and tube heat exchanger might be superior to the other three arrangements.

In the previous example, it was possible to work out the required information directly because three of the four inlet and outlet temperatures were known and the fourth could be computed from an energy balance. If only inlet temperatures were known, then this poses a difficulty to the design process. A method of overcoming this is illustrated in the following example:

Example 5.3

The double pipe heat exchanger of Example 5.2 is to be used to cool 0.15kg/s of oil at 90oC using 0.3kg/s of seawater at 10oC. The area of the heat exchanger is 11.5m2 and the overall heat transfer coefficient is 21.9W/m2K. What are the exit states of oil and water from the heat exchanger?

Solution

To be able to calculate temperatures, an energy balance need to be performed. This requires knowledge of the overall heat transfer Q. This can be computed from the relation:

$$Q = UA\Delta T_{lm}$$

However, ΔT_{lm} is not known so an iterative procedure needs to be followed. The steps are:

Assume a value of $T_{h,o}$

Calculate $T_{c,o}$ from energy balance as in example 5.2

Calculate ΔT_{lm} and then Q

From Q and the capacity rate, compute $T_{h,o}$

Compare $T_{h,o}$ with the initial value. If the two do not agree (say within $\pm 0.5^o C$ repeat the iteration with a new value of $T_{h,o}$ until convergence (usually taking the average between current and previous solutions improves the rate of convergence).

So the sequence becomes:

Assume $T_{h,o} = 70^o C$

$$\dot{m}_h C_{p,h}(T_{h,i} - T_{h,o}) = \dot{m}_c C_{p,c}(T_{c,i} - T_{c,o})$$

$$T_{c,o} = \frac{\dot{m}_h C_{p,h}(T_{h,i} - T_{h,o})}{\dot{m} C_{p,c}} + T_{c,i} = \frac{0.15 \times 2131(90 - 70)}{0.3 \times 4178} + 10 = 15.1^o C$$

$$\Delta T_1 = (90 - 10) = 80^o C$$

$$\Delta T_2 = (50 - 15.1) = 24.9^o C$$

$$\Delta T_{lm} = \frac{24.9 - 80}{\ln(24.9/80)} = 66.66^o C$$

Evaluate Q:

$$Q = UA\Delta T_{lm} = 21.9 \times 11.5 \times 66.66 = 16788\, W$$

Calculate new $T_{h,o}$

$$Q = \dot{m}_h C_{p,h}(T_{h,i} - T_{h,o})$$

$$T_{h,o} = T_{h,i} - \frac{Q}{\dot{m}_h C_{p,h}} = 90 - \frac{16788}{0.15 \times 2131} = 37.5^o C$$

Comparing with assumed value, the difference is 32.5oC

A reasonable choice for the 2nd iteration is the average value between the first assumption and the computed value, therefore, for the 2nd iteration, assume:

$$T_{h,o} = 0.5(70 + 37.5) = 53.7°C$$

From which:

$$T_{c,o} = 19.24°C$$

$$\Delta T_{lm} = 50.09°C$$

$$Q = 13623\,W$$

$$T_{h,o} = 47.38°C$$

The difference now is 6.3oC

For third iteration, take:

$$T_{h,o} = 0.5(53.7 + 47.38) = 50.56°C$$

From which

$$T_{c,o} = 20.05°C$$

$$\Delta T_{lm} = 51.33°C$$

$$Q = 12356\,W$$

$$T_{h,o} = 51.32°C$$

giving a difference of 0.7oC

A fourth iteration is necessary to bring error within specified bound.

5.5 Summary

In this Chapter, we discussed the main classifications of heat exchangers. The chapter then focused on the analysis for recuperation type.

Recuperation heat exchangers were classified based on three criteria. The first is based on flow arrangement, where heat exchangers are classified to parallel flow, counter flow and cross flow. The second criterion was based on construction. Of these, we mentioned double pipe, U-tube and shell and tube heat exchangers. A third classification is that based on compactness where heat exchangers are classified according to volume to surface area ratio.

We then derived a formulation for the overall heat transfer coefficient for straight wall and circular pipe heat exchangers. The overall heat transfer coefficient takes into account the conductance in the two fluids and the metal separating them. The overall heat transfer coefficient U is defined such that the heat transfer is given by:

$$Q = UA\Delta T_m$$

The final stage is to provide analysis of heat exchangers, were the Log Mean Temperature Difference (LMTD) method was used. This led to the formulation for ΔT_m which can be used in the above equation to calculate the heat transfer in a heat exchanger.

5.6 Multiple Choice Assessment

1. Which is NOT an example of a heat exchanger ?
   - automotive radiator
   - central heating radiator
   - electric kettle
   - engine oil cooler
   - cooling tower

2. Which heat exchanger configuration has the highest thermodynamic efficiency ?
   - contra-flow
   - cross flow
   - parallel flow
   - direct contact
   - regenerative

3. A car radiator may be classified as what sort of heat exchanger ?
   - shell and tube
   - plate fin
   - tube fin
   - double pipe
   - direct contact

4. For a heat exchanger passage with a value of heat transfer coefficient on each side, h, the overall heat transfer coefficient is:
   - 2h
   - h2
   - (h)1/2
   - h/4
   - h/2

5. A typical value for the overall heat transfer coefficient in an air-to-air heat exchanger would be:
   - 2 W/m2 K
   - 20 W/m2 K
   - 200 W/m2 K
   - 2000 W/m2 K
   - 20000 W/m2 K

6. A heat exchanger passage has a rectangular cross-section of sides a and b, what is the hydraulic diameter ?
   - (a2 + b2) / (a + b)
   - (a + b) / 2
   - 2ab / (a + b)
   - a
   - b

7. To increase the overall heat transfer coefficient in an air to water heat exchanger one would:
   - increase the flow rate of the water
   - increase the flow rate of the air AND the water
   - increase the flow rate of the air
   - increase the air pressure
   - none of these

8. The logarithmic mean temperature difference is defined as:
   - (ΔT2 – ΔT1) / loge (ΔT1 / ΔT2)
   - ΔT2 – ΔT1
   - loge (ΔT1 / ΔT2)
   - (ΔT1 – ΔT2) / loge (ΔT1 / ΔT2)
   - (T2 – T1) / loge (ΔT1 / ΔT2)

9. A heat exchanger is used to cool 1 kg / s of oil (Cp = 2 kJ / kg K) from 90˚C to 70˚C with a 0.5 kg / s flow of water (Cp = 4 kJ / kg K). If the water has an inlet temperature of 10˚C, what is the water exit temperature ?
   - 10˚C
   - 20˚C
   - 30˚C
   - 40˚C
   - 50˚C

10. Evaluate the logarithmic mean temperature difference for the heat exchanger in Question 10 in parallel flow
    - 94.4˚C
    - 18.2˚C
    - 40˚C
    - 57.7˚C
    - -22 ˚C

11. As question 11) but for contra-flow
    - 40˚C
    - 50˚C
    - 60˚C
    - 70˚C
    - 80˚C

12. If the overall heat transfer coefficient (of the heat exchanger flows in Question 10) is 300 W / m2 K, what is the surface area required in parallel flow ?
    - 2.3 m2
    - 5 m2
    - 6.7 m2
    - 8 m2
    - 9.2 m2

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